Supplementary Section 6S.9

Logical Truth, Analyticity, and Modality

In chapter 2 of *Introduction to Formal Logic with Philosophical Applications*, we classified propositions as tautologies, contingencies, or contradictions. The tautologies were especially interesting to identify because they are the logical truths of **PL**. In chapter 3, we saw how to use derivations to show that a proposition is a logical truth. In section 4.6, we saw how to prove logical truths of **M**; in section 5.3, we saw how to prove logical truths of **F**. In this section, we will characterize the nature and importance of the concept of logical truth.

Logical truths are privileged sentences of logical systems, ones that are true on all interpretations. In an axiomatic system, like formal treatments of Euclidean geometry or Newtonian mechanics, we choose a small set of sentences that we call axioms, and that define the system. In some cases, we insist that the axioms be obvious and uncontroversial. More often, they are chosen to be the weakest claims from which the ones we wish to prove, the theorems, follow. The theorems of a formal system are the statements that are either axioms or provable from the axioms. Some sentences of propositional logic are theorems. These statements are the logical truths.

We identify any formal system with its logical truths. Two or more systems may have different axioms, but prove the same theorems; we call such theories equivalent. Competing theories have different theorems. Thus, knowing the logical truths of a system of logic is central to understanding the logic.

To get a feel for the nature of logical truths, compare 6S.9.1, 6S.9.2, and 6S.9.3.

6S.9.1 If it is raining, then I will be unhappy.
6S.9.2 If it is raining, then I will get wet.
6S.9.3 If it is raining, then it is raining.

Each of the three sentences is expressible in **PL** as 'P ⊃ Q'. But 6S.9.1 and 6S.9.2 are contingent sentences. The truth of 6S.9.2 is more compelling, but it is still possible for both sentences to be false. 6S.9.3, on the other hand, can never be false, as long as we hold the meanings of the terms constant. It is more carefully regimented as 'P ⊃ P', and it is a logical truth, or a law of logic.

LOGICAL TRUTH AND NECESSITY

The concept of logical truth is, on some interpretations, closely related to the concept of necessity. For **PL**, the tautologies are necessarily true, true for all values of the component propositions; there are no cases in which a tautology is false. The contradictions, negations of tautologies, are necessarily false. In quantificational logic, the logical truths are propositions that are true for all interpretations of the predicates and constants.

The concepts of necessary truth, necessary falsehood, and contingency have some intuitive meanings in ordinary language. Within formal logic, their closest correlates are the concepts of logical truth, logical falsehood, and logical contingency. Formal logic may usefully illuminate those intuitive meanings, though we cannot capture everything we mean in ordinary language with those formal terms.

Necessity is easily defined in terms of possibility. A proposition is necessarily true if it is not possible for it to be false. A proposition is possibly true if it is not necessarily false. A proposition is contingent if it is possibly true, but not necessarily true. But these characterizations just show the interrelation of the terms 'necessary' and 'possible' without characterizing them fully. What does it mean for a proposition to be necessary or possible?

To get a sense of what we mean by the terms 'necessary' and 'possible', consider 6S.9.4–6S.9.7.

6S.9.4 Aristotle distinguished four kinds of causes.
6S.9.5 Descartes defended mind-body materialism.
6S.9.6 $2 + 2 = 4$
6S.9.7 $2 + 2 = 5$

6S.9.4 and 6S.9.6 are true; 6S.9.5 and 6S.9.7 are false. 6S.9.6 is often taken to be necessarily true, whereas 6S.9.4 is usually seen as merely contingently true. It is not possible for two and two to be anything other than four, as long as we hold the meanings of these terms constant, and as intended. But Aristotle could, we imagine, have distinguished a fifth kind of cause, or only three. Similarly, 6S.9.5 is usually called contingently false, since Descartes might have defended materialism rather than dualism. But 6S.9.7 is necessarily false.

To complicate matters a bit, there also seem to be different kinds of necessity. A statement might be physically necessary if it follows from the laws of physics. Objects near the surface of Earth accelerate, inexorably, toward the ground at about 9.8 m/s^2 if not opposed by a supporting force. But we can imagine the physical laws being different from what they are, that there are at least possible worlds in which gravitation acts differently from the way it does in our universe.

We capture the difference between the way things must be in our universe and the way they might be in other possible worlds using the concept of metaphysical necessity. A statement is said to be metaphysically necessary if it is true in all possible worlds, even those in which the laws of physics are different. A metaphysically

possible statement is one that is true in some possible worlds, and a metaphysical impossibility is one which is true in no possible worlds.

Philosophers talk about possible worlds as a way of talking about other ways this world could be. I am, let's say, wearing a blue shirt today. I could have put on a black shirt. In the actual world, my shirt is blue, but in another possible world, my shirt is black. It is difficult to know exactly how we acquire and justify beliefs about other possible worlds. Our beliefs about the actual world come, at least in part, from sense experience: I look at or hear or smell the world. Our beliefs about other possible worlds come from thinking about how this world might be different. We can imagine, to some extent, different physical laws. But it does not obviously follow from our ability to imagine the world as different that the world could be different in any practical or concrete or useful sense. Thus the concept of metaphysical necessity is contentious.

In addition to physical and metaphysical necessity, philosophers sometimes talk about mathematical necessity. A statement is mathematically necessary if it follows from axioms of mathematics. Most, if not all, mathematical theorems appear to be necessary in some sense. It seems possible that Descartes could have defended mind-body materialism in a way that it seems impossible for two and two to be five. But again, it is difficult to explain exactly what it means for a statement to be mathematically necessary, especially if one asks the sticky question of how one decides on the axioms of mathematics. Further, it is hard to know how mathematical necessity relates to physical or metaphysical necessity.

Let's put mathematical, metaphysical, and physical necessity aside and return to our subject of logical necessity, and its related concepts of tautology, contingency, and contradiction.

VARIETIES OF LOGICAL TRUTH

The formal logical tools we use to explicate logical necessity and its related concepts seems to make those concepts broad and clear. Our tools avoid philosophical concerns about our access to possible worlds or the physical laws. They are specifiable often in effective ways.

Still, there are a variety of different characterizations of logical truth. Some variety comes from the different logical systems. The logical truths of **PL**, characterized by the tautologies, are different from the logical truths of **M**, or **F**, which require the semantics of predicate logic. In **PL**, the logical truths concern relations among complete propositions. In predicate logic, the logical truths concern relations among parts of propositions: singular terms and predicates (or the objects for which they stand). But within the logical systems discussed in this book, the logical truths are neatly specifiable.

Some of the diversity of characterizations of logical truth comes from different ways of thinking about logical truth. We might think of the logical truths as those that have a certain logical structure, like 'if it is raining, then it is raining'. Any statement of the

form 'If α then α' will be a logical truth. In parallel, for predicate logic, 'if something bears some relation to everything then everything is related in that way to something' is a logical truth; propositions of that structure will hold for any relation. Relatedly, we can characterize logical truths as those that are true no matter the content of its simple components, no matter what we put for α, say, in 'If α then α'. The former characterization focuses on the structure of the logical truth; the latter on the variations in content.

We can also think of the logical truths as those that are provable within a logical system. For **PL**, the provable wffs are the same as the tautologies. All logical truths of **M** and **F** are also provable within our derivation systems for those languages. But in some formal systems, there are truths that are unprovable. So one must be careful when invoking proof to characterize logical truth.

Summary

Logical truths are the characteristic claims of a logical theory, the ways in which we identify and distinguish different logical theories. The logical truths of **PL** are characterized differently from those of **M** and those of **F**. What is common to the definitions of logical truth in various theories is the thinness of the logical claim and, generally, the lack of existential implication: logic doesn't say anything about the world and is compatible with all sorts of ways in which the world might be. We might call such claims necessary truths, so an understanding of the different kinds of necessity may illuminate your understanding of logical truths. But be aware: some philosophers, most notably Wittgenstein, believe that calling logical truths necessary imbues them with too much weight; instead, he calls them nonsense.

TELL ME MORE

- How do logicians formalize the concept of necessity? See 6.5: Modal Logics.

For Further Research and Writing

1. Describe and distinguish the four different kinds of necessity (metaphysical, physical, mathematical, and logical) and their related concepts of contingency and necessary falsehood. How do the other concepts help illuminate the nature of logical truth? Is the concept of logical necessity helpful in illuminating the others?
2. What are the different ways to characterize logical truth? What are their strengths and weaknesses? See especially Quine's *Philosophy of Logic*, chapter 4; Fisher's *On the Philosophy of Logic*, chapter 9; and Sainsbury's *Logical Forms*, section 6.5.

3. Many theories use axioms. Even some logical theories use axioms; see section 6S.11: Axiomatic Systems. The logical theories in this book generally do not. Explain how they can avoid axioms. How can we identify our theory without using axioms?
4. At the end of this section, I noted that in some theories, there are statements that are true but unprovable. Is this true about logical theories? Are there logical truths that cannot be proven? Rebecca Goldstein's book on Gödel's theorems is a mainly gentle introduction. See Nagel and Newman's classic *Gödel's Proof* for a bit more rigor. Hofstadter's Pulitzer Prize–winning *Gödel, Escher, Bach: An Eternal Golden Braid* is an ambitious riff on themes from Gödel's techniques in music, art, and computer science. See the discussion of completeness and other results in section 6.4: Metalogic, and the references there.
5. Are logical truths necessary or nonsense? What it would it mean to call them nonsense? One of Wittgenstein's early works, the *Tractatus Logico-Philosophicus*, is the locus for the view that logical truths are nonsense. Two articles in the *Cambridge Companion to Wittgenstein* ("Pictures, Logic, and the Limits of Sense in Wittgenstein's *Tractatus*" by Thomas Ricketts and "Necessity and Normativity" by Hans-Johann Glock) might be helpful. G. E. M. Anscombe's classic *An Introduction to Wittgenstein's Tractatus* is an excellent introduction to Wittgenstein's early work.

Suggested Readings

Anscombe, G. E. M. *An Introduction to Wittgenstein's* Tractatus. An excellent, accessible introduction to Wittgenstein's early work, in which he characterizes logical truths as nonsense.

Fisher, Jennifer. *On the Philosophy of Logic*. Belmont, CA: Wadsworth, 2008. Chapter 9 discusses the problem of logical truth explicitly, but chapter 7, on bivalence, is also useful.

Glock, Hans-Johann. "Necessity and Normativity." In *The Cambridge Companion to Wittgenstein*, edited by Hans Sluga and David G. Stern, 198–225. Cambridge: Cambridge University Press, 1996. Glock discusses Wittgenstein's views about logical truth and compares them to those of the logical empiricists more generally and Quine.

Goldstein, Rebecca. *Incompleteness: The Proof and Paradox of Kurt Gödel*. New York: Norton, 2005.

Haack, Susan. *Philosophy of Logics*. Cambridge: Cambridge University Press, 1978. Chapter 2, on validity, is clear and useful, and has further excellent references.

Hofstadter, Douglas. *Gödel, Escher, Bach: An Eternal Golden Braid*. New York: Basic Books, 1999.

Kneale, W., and M. Kneale. *The Development of Logic*. Oxford, UK: Clarendon Press, 1962. In this classic history of logic, the first and fifth sections of chapter 10, "The Philosophy of Logic After Frege," could be useful.

Kripke, Saul. *Naming and Necessity*. Cambridge, MA: Harvard University Press, 1980. An engaging and ranging discussion, rooted in questions about names. Kripke's discussions of necessity are particularly useful.

Nagel, Ernest, and James R. Newman. *Gödel's Proof*, rev. ed. New York: New York University Press, 2001.

Pap, Arthur. "The Laws of Logic." In *A Companion to Philosophical Logic*, edited by Dale Jacquette 13–20. Malden, MA: Blackwell, 2002. A clear discussion of the status of tautologies.

Quine, Willard van Orman. *Philosophy of Logic*, 2nd ed. Cambridge, MA: Harvard University Press, 1986. Chapter 4 lays out an intriguing variety of definitions of logical truths.

Read, Stephen. *Thinking About Logic*. Oxford: Oxford University Press, 1995. Chapter 2, on logical consequence, is accessible and has lots of great further references.

Ricketts, Thomas. "Pictures, Logic, and the Limits of Sense in Wittgenstein's *Tractatus*." In *The Cambridge Companion to Wittgenstein*, edited by Hans Sluga and David G. Stern, 59–99. Cambridge: Cambridge University Press, 1996. Ricketts provides a nice overview of Wittgenstein's early work, especially focusing on how the picture theory treats logic.

Sainsbury, Mark. *Logical Forms: An Introduction to Philosophical Logic*, 2nd ed. Oxford, UK: Blackwell, 2001. The first chapter, on validity, is engaging, with exercises. Chapter 6, section 5, engages Quine's various views of logical truth.

Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus.* Translated by C. K. Ogden. London: Routledge, (1922) 1990. The source of the claim that logical truths are nonsense. Anscombe's guide is a good introduction to this often cryptic essay. The essays from Glock and Ricketts listed above might be useful and to the point here.