Supplementary Section 7S.11

Atomism and Color Incompatibility

The question of the scope and importance of deductive logic is a perennial topic for philosophers of logic. Some philosophers see logic as a severely limited discipline, governing only the most obvious and incontrovertible inferences. Others see it as the foundation for all human reasoning, a normative discipline prescribing the ways in which rational beings should think. The beliefs of most philosophers nowadays lie somewhere between these two extremes. So, a central question for philosophers concerns the importance of the formal definitions of logical consequence that are developed in the first five chapters of *Introduction to Formal Logic with Philosophical Applications* for human reasoning more widely: why do we study logic?

Kant, in the late eighteenth century, had seen logic as a closed and complete discipline; see section 1.3 of *IFLPA*. But Frege's new mathematical logic, developed a century later, was remarkably more powerful, especially in its use of quantifiers and its general treatment of relations. The new logic was so successful in unifying propositional logic with term logic and generalizing their study that some philosophers initially supported hopes of it being the canonical language of all of knowledge. Through the twentieth century, Quine was a proponent of that view, and some philosophers continue to believe that first-order logic is canonical.

An early application of Frege's logic to broader philosophical purposes came from Ludwig Wittgenstein, in his 1919 *Tractatus Logico-Philosophicus*. Wittgenstein had visited Frege in Germany as a young student and had studied with Bertrand Russell, before World War I, in Cambridge, England. He worked on the *Tractatus* during his subsequent time in the Austrian army during the war. His treatise was published after the war, with an introduction by Russell. Russell described Wittgenstein's *Tractatus* as the culmination of the enterprise of logical analysis begun by Frege.

WITTGENSTEIN'S LOGICAL ATOMISM

According to the *Tractatus*, the world is a collection of independent atomic facts combined according to logical principles. If we could get clear about the correct logic,

Wittgenstein argued, then we could have a complete, accurate picture of the world in our best, most austere language.

The *Tractatus* was highly influential in Europe between World War I and World War II, as the foundation of logical empiricism, or logical positivism. A group of logical empiricist philosophers influenced by the *Tractatus*, including Rudolf Carnap, Otto Neurath, Moritz Schlick, Carl Hempel, and Herbert Feigl, came to be known as the Vienna Circle. A less-influential group called the Berlin Circle was centered around physicist Hans Reichenbach. The young A. J. Ayer visited Vienna from England in the early 1930s and wrote about the movement. Ayer's *Language, Truth, and Logic* became a primary source for logical empiricism for English-speaking philosophers.

One could easily spend an entire term studying the *Tractatus*, let alone logical empiricism. *The Tractatus* is obscure when read directly, consisting of a series of numbered aphorisms. There are seven main propositions, and all but the seventh have sets of explanatory subpropositions. Wittgenstein seeks the limits of language in distinguishing between what can and what cannot be said.

> §7. Whereof one cannot speak, thereof one must be silent.

The project of distinguishing between what can and cannot be said, or between what can and cannot be thought, naturally meets a fundamental difficulty. If we want to distinguish between, say, the backyards of two houses, we can draw a boundary line. We perceive both sides of the line, and see the landscape divided. This side belongs to the Majors; this other side belongs to the Stockwells. In contrast, attempts to draw a line between what is expressible in language and what is not expressible are essentially more problematic. What is outside of the scope of language is inexpressible. What is outside the boundary of thought cannot be thought. We can look at both sides of a fence. We can talk about and think about only one side of the boundaries of language and thought.

Still, Wittgenstein believed that we can at least try to get clear about how our language functions and what its limits are. If we cannot describe what is outside the limits of language, at least we can bump up against the edges.

The *Tractatus* presents an atomistic picture theory of meaning on which language mirrors the world. The world, Wittgenstein alleges, is a collection of independent states of affairs. Suppose that I am standing to the right of you. We have, let's say, two atomic facts, my standing and your standing, and a logical relation, being to the right of, which holds between those facts. I could stand to the right of you, or to the left of you, or on the other side of the planet. All of my relations to you are independent of you.

> §1.2. The world divides into facts.
> §2.06. From the existence or non-existence of one state of affairs, it is impossible to infer the existence or non-existence of another.

On Wittgenstein's view, language consists of atomic statements of those facts, connected into more complex statements by logical principles. The structure both of language and of the world is governed by logical rules, like those depicted in the truth

tables. Indeed, Wittgenstein was the first to develop truth tables, in the *Tractatus*; see section 5.31. Language mirrors the world by providing a logical structure that is structurally equivalent, or isomorphic, to the structure of the world.

> §2.16. If a fact is to be a picture, it must have something in common with what it depicts.
> §2.17. What a picture must have in common with reality, in order to be able to depict it—correctly or incorrectly—in the way it does, is its pictorial form.

Since language and logic have the same form as the world, we can know about the fundamental structure of reality by examining the fundamental structures of language and logic.

Of course, we cannot rely on the surface grammar of natural language to reflect the structure of the world. Natural language is sloppy, full of misleading metaphors and pragmatic shorthand. If we want a true representation of the world, we must seek a finer language, like Frege's mathematical logic. Recall Frege's claim, which we saw in section 1.2 of *Introduction to Formal Logic with Philosophical Applications*, that his *Begriffsschrift* is like a microscope on our language. Wittgenstein believed that Frege's logic is the precision tool that the picture theory requires to represent the atomic facts of the world, and to show how they are related and combined. The correct logic will mirror the structure of the world. The correct logic, therefore, is essential to a proper understanding the nature of reality.

To see how the demands for precision are manifested, notice that my example of an atomic fact, my standing to the right of you, is misleading. My standing in a place is not an atomic fact; it is a complex fact. Complex facts are those that are analyzable into more-fundamental facts. Your body and mine are both complex, since they are divisible into smaller parts. Standing is also a complex, since it is divisible into more fundamental facts about the position of our bodies. The true analysis of the world involves analyzing such complexes into their simple, atomic components.

Atomic facts are the foundational elements for the *Tractatus*, akin to the axioms of Euclidean geometry, say, or to Descartes's *cogito*. Wittgenstein's goal, in the *Tractatus*, was a theory of the world that analyzes all of the myriad complexes into their atomic elements. Such a theory would present a veridical and secure picture of the world. If we got the atomic elements right and combined them into the correct logic, our theory of the world would mirror the world precisely. We would have the isomorphism between language and the world that we want.

Because of its method of analyzing complex propositions into elementary ones, the kind of philosophy that was developed by the early Wittgenstein, under the influence of Frege and Russell, was called analytic philosophy. The name 'analytic philosophy' remains as a characterization of Anglo-American philosophy, despite the lack of contemporary interest in the project of analysis in this sense. But Wittgenstein's original plan was to use the new logic, because of its utility for analysis, to represent the atomic facts of the world in elementary propositions and their logical combinations.

THE PROBLEM

The problems facing atomism and logical empiricism arise, in the *Tractatus*, in Wittgenstein's worry whether there are independent atomic facts. Atomic facts are supposed to be most basic, not analyzable into further simple facts. Facts about our bodies, we saw, are not atomic because they can be reduced to facts about parts of our bodies. Properties like standing are not atomic for the same reason. It is a challenge to try to think about what kinds of facts could be most fundamental, irreducible to other facts.

Wittgenstein never gives a clear example of an atomic fact. Russell used the example of the color of a spot in my field of vision. A dot in one's field of vision seems as likely a candidate for an atomic fact as any for several kinds of reasons. First, a small dot of color seems irreducible to other facts. Second, atomic facts are supposed, by definition, to be independent of each other. The color of one dot in my field of vision can be any color, independent of the color of any other spot in my field of vision. Last, however we construct our theories of the world, however complex we believe the world to be, the ultimate arbiter of those theories seems to be our sense experience, like the experience of the color of a spot in our field of vision. Differences in colors in our fields of vision allow us to read the scale of a thermometer, the position of the stars seen in a telescope, and the motion of an object traveling toward us. Related facts which also seem simple include auditory tones and odors and tastes.

But since sight is, for most of us, the most fecund of the senses, let's stick to the color of a spot in our field of vision. Wittgenstein noticed that even such simple facts cannot be atomic because they are not independent. Instead, they carry some sorts of entailment relations.

> §6.3751. It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colors at the same time is a contradiction.

Spots in one's color field seem paradigmatically atomic. Atomic facts must be independent. But spots in our color field are not independent.

Jerrold Katz, in "The Problem in Twentieth-Century Philosophy," characterizes Wittgenstein's 6.3751 as the central problem in analytic philosophy. To explicate the problem, he considers six sentences.

7S.11.1 The spot is red and blue.
7S.11.2 The spot is red.
7S.11.3 The spot is not blue.
7S.11.4 The spot has a color.
7S.11.5 Red is a color.
7S.11.6 The spot is green.

7S.11.2, 7S.11.4, 7S.11.5, and 7S.11.6 might be supposed to express atomic facts; 7S.11.1 and 7S.11.3 are supposed to be simple logical products of elementary propositions.

But 7S.11.1 is a contradiction. 7S.11.2 and 7S.11.5 are incompatible, and 7S.11.2 entails 7S.11.3 and 7S.11.4. There are substantial logical relations among these propositions even though they appear to be elementary. If such facts are not atomic, then it is hard to see how any facts could be atomic. The world appears not to be atomic in the way that the *Tractatus* depicts.

If the elementary propositions are interdependent, it is difficult to see how they could serve as the foundations of other beliefs. If the proposition that this spot is green entails that it is not red, and not purple, and that it is a color, and that spots are incompatible with each other, and so on, I cannot just immediately and securely know a single, simple fact. Such claims would be comprehensible only en masse.

The problem of how to understand how elementary propositions can have logical relations among them has become known as the color incompatibility, or color exclusion, problem. As Katz observes, the problem is not merely about color.

> It is a general problem about the extralogical vocabulary of the language and about all the semantic properties and relations of the language. (Katz, "Problem," 548)

The problem can be seen in any sentence whose truth seems to be both logical and dependent on the meanings of terms. 7S.11.7 appears to be a special kind of sentence, one whose truth is guaranteed by its meaning, like a logical truth.

7S.11.7 Bachelors are single.

We can regiment 7S.11.7 into predicate logic, as 7S.11.8.

7S.11.8 $(\forall x)(Bx \supset Sx)$

The logic of 7S.11.8 does not reveal the special status of 7S.11.7. There are logical relations among the terms 'bachelor' and 'single'. But the logic we have been studying does not show those relations. The atomists, including Wittgenstein and the logical empiricists who followed him, could not accommodate the relationship between various atomic facts in their logic.

MEANING POSTULATES

It might seem rather easy to treat the color incompatibility problem. We can just adopt statements like 7S.11.9–7S.11.11 as axioms.

7S.11.9 All bachelors are unmarried.
7S.11.10 Red is a color.
7S.11.11 Red is not blue.

This proposal was explored by Rudolf Carnap, one of the more prominent of the logical empiricists. Propositions like 7S.11.9–7S.11.11 are extralogical; they are about meanings rather than about logic. Carnap's proposal is that we can stipulate whatever meaning relations we believe to be important.

The stipulation involved in adopting meaning postulates leads to two serious problems. First, we would have to adopt a lot of meaning postulates. Red is not blue, and

not green, and not a ball of feta cheese, and not the Archbishop of Canterbury. It is not plausible that we believe in any conscious way all of the required meaning postulates.

Second, and more problematic, a long list of meaning postulates is not an explanation of why such postulates hold.

> Meaning postulates serve as constraints on the assignment of extensions to sentences, but they cannot explain the property common to the sentences they enumerate. Like Socrates's interlocutors, meaning postulates offer examples of the concept instead of the concept. (Katz, "Problem," 553)

This second problem with meaning postulates is subtle, so let's take a moment to spell it out carefully. If I stipulate that no Ps or Qs are Rs, then it will follow that no Ps are Rs. We could adopt as an axiom the meaning postulate 7S.11.12.

7S.11.12 $(\forall x)[(Px \vee Qx) \supset \sim Rx)] \supset (\forall x)(Px \supset \sim Rx)$

7S.11.12 is a logical truth of **F**. But 7S.11.12 holds for any values of P, Q, and R. And 7S.11.12 does not tell us anything about the relationship between Ps and Rs. It does not tell us that there is a relationship between being a P that entails being an R. It says, for example, that if all blue or green things are not red, then all blue things are not red. But we want an explanation of the relationship between blue and not red. We want an explanation of the consequent of 7S.11.12, not merely that it follows from its antecedent as a logical truth.

Compare 7S.11.12 with 7S.11.13, which is a logical truth of **PL**.

7S.11.13 $(P \supset Q) \supset [(Q \supset R) \supset (P \supset R)]$

7S.11.13, like 7S.11.12, is not the result of any stipulation. It is a theorem of our logic. If meaning postulates were able to do the work that Carnap wants them to do, they would give the status that 7S.11.13 has, and that the entire 7S.11.12 has, to just the consequent of 7S.11.12.

Using meaning postulates to solve the color incompatibility problem makes sentences 7S.11.1–7S.11.6 true by stipulation. But we can stipulate anything we like. We can adopt scientific postulates about the world. We can also adopt axioms governing fictional worlds. Our use of logic within a system of postulates does not determine the truth of those postulates. We want the truth of propositions 7S.11.1–7S.11.6 to be true as a matter of the logic of the terms, like 7S.11.12 and 7S.11.13, rather than as a matter of stipulation, like the consequent of 7S.11.12.

SEMANTIC MARKERS

In order to avoid the problems with Carnapian meaning postulates, Katz proposes a constraint on any solution of the color incompatibility problem.

> A new way out must reject Carnap's assumption that the external, logical structure of extralogical words is the source of analyticity, contradiction,

> and analytic entailment in connection with sentences like [7S.11.1–7S.11.6]. It must assume instead that such properties and relations derive from the internal, sense structure of extralogical words. (Katz, "Problem," 553)

Katz proposes that in addition to the mathematical logic of Frege, we need a formal theory of semantic entailment, one that gets to the analyticity of meanings. Just as we went beneath the level of the sentence moving from **PL** to **M**, we can move beneath the level of logical form to semantic form.

Katz calls the semantic structural properties of syntactically simple terms (like color terms) decompositional sense structure. Senses are meanings. Decompositional sense structure is not syntactic. It depends essentially on meanings, and not the forms of terms. A term like 'bachelor', which is syntactically simple, can be semantically complex.

> The sense of 'single man' is complex, being a compositional function of the senses of 'single' and 'man'. Since 'single man' and 'bachelor' have the same sense, the sense of 'bachelor' is complex. (Katz, "Problem," 555)

Decompositional sense structure is not logical, as the color incompatibility problem shows. The consequent of 7S.11.12 is nothing like a logical truth. It is undeniably true on the given interpretation; wholly green things cannot be red. But the nonredness of something, while derivable from its greenness, is not a logical entailment. It is a semantic entailment.

In order to formalize the notion of semantic entailment, Katz introduces a technical device he calls semantic markers. Semantic markers allow us to analyze concepts, like of being particular color, in such a way as to reveal the entailments like the ones expressed in 7S.11.1–7S.11.6. I will not pursue the complex details of Katz's device here, and a full theory of semantic markers has not been developed.

Katz uses semantic markers to represent the decompositional sense structure of what appeared to Wittgenstein to be elementary propositions. 'This spot is blue' is not a semantically elementary proposition; it presupposes a variety of analytic entailments. On Katz's analysis, blueness can still be a primitive sense in that it is not definable in terms of other senses. But the primitiveness of the sense does not entail that it is semantically simple. It has analytic relations with other senses, despite being primitive. Katz calls the senses of basic color terms complex primitive senses. They are primitive in that they are not reducible to other senses. They are complex, since they have semantic relations to other senses. Senses are thus both inside and outside of logic. Sense entailments are additional to logical ones.

But they constrain logic, since they guide entailments.

> Since senses provide the fine-grained linguistic structure necessary for a model-theoretic explanation of why such sentences have such logical properties and relations, senses are inside logic in precisely Wittgenstein's sense of "hav[ing] an effect on one proposition's following from another." (Katz, "Problem," 572)

Katz's semantic markers have not caught on with philosophers. While they are patterned after Noam Chomsky's syntactic theories of language, they are much more contentious. Many philosophers are wary of meanings. Senses are objective in that they transcend any particular thinker or language user. But they are abstract objects, not the kinds of things that we can perceive with our senses. Thus, some philosophers think of them as spooky entities. Still, senses give us a way of understanding the semantic relations among terms without abandoning Wittgenstein's atomism.

The more popular response to Wittgenstein's problem is holistic, abandoning the atomism of the logical empiricists and Katz's concept of semantic primitives. Many of the more prominent holists, like Quine, also deny the existence of meanings.

LOGICAL EMPIRICISM, SENSE, AND NONSENSE

Color incompatibility is a puzzle for both Wittgensteinian atomists and the logical empiricists who followed Wittgenstein because it looks as if there is a logical relationship between various atomic facts. To see how the problem manifests itself for logical empiricism, we need to look more closely at the broader aims of that philosophical movement.

The logical empiricists saw Wittgenstein's picture theory as accommodating a scientific view of the world. Scientific laws, for example, were seen as mere generalizations over, and reducible to, the separable atomic facts. The logical empiricists believed that all our legitimate claims could be traced to a core set of simple observations.

> There is a class of empirical propositions of which it is permissible to say that they can be verified conclusively. It is characteristic of these propositions, which I have elsewhere called "basic propositions," that they refer solely to the content of a single experience, and what may be said to verify them conclusively is the occurrence of the experience to which they uniquely refer. . . . Propositions of this kind are "incorrigible," . . . [in that] it is impossible to be mistaken about them except in a verbal sense. (Ayer, *Language Truth and Logic*, 10)

The logical empiricists claimed that all of science and philosophy could be founded on the basis of observation statements in conjunction with the logical and mathematical principles used to regiment and derive those observations. Claims that are not observable may be derived from the axiomatic observations or introduced by definition. Lastly, some claims, like logical truths, are neither observable nor derivable from observable claims. Hume called such claims relations of ideas. The logical empiricists called them analytic truths. Among the analytic truths were supposed to be logical truths and, for logicists like Frege and Russell, the truths of arithmetic. For the logical empiricists, all and only meaningful statements will be analytic, or observable, or derivable (using logic) from observable axioms.

A fundamental presupposition of logical empiricism, then, is that one can make a clear distinction between an observation statement and an analytic one. This

distinction was rooted in Wittgenstein's distinction between sensible statements and logical nonsense. Let's take a moment to look at that distinction.

One of the most important advances in modern logic was its ability to characterize a broad, general concept of logical truth. Logical truths of **PL** are tautologies, complex statements that are true no matter the truth values of their component variables. Logical truths of **F** are true on any interpretation.

We might characterize logical truths as necessary truths. Descartes, for example, believed that the certainty of logic and mathematics provided essential support to his claim that our minds have substantial content built into their structures. From the claim that logic and mathematics are innate, it is reasonable to ask whether there are other innate ideas, including the idea of God.

Wittgenstein thought that characterizing logical truths as necessary imbues them with too much importance. In contrast, he called them nonsense. Only statements that can picture the world have sense. Only such statements can be either true or false and can picture accurately or not. Tautologies are empty of content.

> §4.461. The proposition shows what it says, the tautology and the contradiction that they say nothing. The tautology has no truth conditions, for it is unconditionally true; and the contradiction is on no condition true. Tautology and contradiction are without sense.
> §6.1251. Hence, there can *never* be surprises in logic.

Logical truths are unknowable because they are too thin to be objects of knowledge. They don't picture any fact. Wittgenstein wanted carefully to circumscribe what we can know.

The logical truths were, for Wittgenstein, logical nonsense. The logical empiricists called them merely analytic. All agreed that they were easily derivable within formal logic. Analytic truths were sharply contrasted with synthetic ones, which had to trace back, or reduce, in some way, to observation. Indeed, the whole of the atomist movement, from Locke and Hume through Wittgenstein and the logical empiricists, rests on this distinction between analytic and synthetic propositions.

HOLISM AND THE ANALYTIC/SYNTHETIC DISTINCTION

Throughout his work, but especially in his seminal paper "Two Dogmas of Empiricism," Quine attacked the logical empiricist's distinction between analytic and synthetic statements and argued instead for holism. Holism is the denial of atomism. The holist claims that there are no individual statements independent of larger theories. Just as the color facts, 7S.11.1–7S.11.6, are not independent, all claims are interrelated.

> Our statements about the external world face the tribunal of sense experience not individually but only as a corporate body. (Quine, "Two Dogmas of Empiricism," 41)

Where the atomist like Wittgenstein applies Frege's logic to atomic, elementary propositions, the holist despairs of finding any simple facts. The holist denies that there is any real difference between analytic and synthetic claims, between truths of logic and empirical truths.

> It is obvious that truth in general depends on both language and extralinguistic fact. The statement "Brutus killed Caesar" would be false if the world had been different in certain ways, but it would also be false if the word "killed" happened rather to have the sense of "begat." Hence, the temptation to suppose in general that the truth of a statement is somehow analyzable into a linguistic component and a factual component. Given this supposition, it next seems reasonable that in some statements the factual component should be null; and these are the analytic statements. But, for all its a priori reasonableness, a boundary between analytic and synthetic statements simply has not been drawn. (Quine, "Two Dogmas," 70)

Our knowledge of synthetic propositions is supposed to be rooted in our sense experience of particular facts. But the particular beliefs that are supposed to be the starting points of our knowledge, the foundations, seem not to be independent. That is a lesson of the color incompatibility problem. Knowledge of purportedly atomic facts seems to require, or presuppose, the understanding of a whole battery of other facts that come along with them. Knowledge that this spot is green entails knowledge that green is a color, that this spot is not red, and so on. This problem seems to undermine the claim that any atomic fact is given, as a foundational belief. If the basic facts are interconnected, they could not possibly be immediately perceivable. They would be comprehensible only as whole systems of claims, a larger theory, a corporate body.

This problem with the analytic/synthetic distinction, call it the holistic insight, is related to the interconnectedness of individual statements we saw in the color incompatibility problem. Individual statements depend for their truth on a broader theory, in contrast to Wittgenstein's atomism. Hempel, another prominent logical empiricist, applied the holistic insight to his account of scientific reasoning.

> In the language of science, and for similar reasons even in prescientific discourse, a single statement usually has no experiential implications. A single sentence in a scientific theory does not, as a rule, entail any observations sentences; consequences asserting the occurrence of certain observable phenomena can be derived from it only by conjoining it with a set of other, subsidiary, hypotheses. ("Empiricist Criteria of Cognitive Significance: Problems and Changes," 56)

Wittgenstein and the logical empiricists presented a system on which individual sentences, pictures of states of affairs, were verified or disconfirmed on their own. Then, they could be connected by logic into a larger theory. The holist's claim is that the meaning of a single expression is elliptical, incomplete on its own. It requires, for its meaning, reference to an entire linguistic framework, a theoretical context that forms the background to that expression.

> If . . . cognitive significance can be attributed to anything, then only to entire theoretical systems formulated in a language with a well-determined structure. (Hempel, "Empiricist Criteria," 57)

Hempel here alludes to what has come to be known as semantic holism: the unit of empirical significance is not the individual sentence, but the entire theory. Holism comes in a variety of forms. Most strong, semantic holism claims that the meaning of any term or sentence depends on the meanings of all of our sentences. Meaning is a property of an entire language, not of individual terms. Less contentiously, confirmation holism claims that individual sentences are confirmed or refuted only by whole theories, not individually. Confirmation holism is a logical fact about sets of sentences. Even two contradictory sentences are compatible in the absence of a larger theory that prohibits contradiction.

Quine holds both the stronger semantic holism and the less-contentious confirmation holism. Wilfrid Sellars argues that the holistic conclusion is not merely about colors, and observation reports of them.

> It follows, as a matter of simple logic, that one couldn't have observational knowledge of *any* fact unless one knew many *other* things as well. (Sellars, "Does Empirical Knowledge Have a Foundation?" 123)

If holism, even in its weak form, is correct, then the presupposition of atomism that some of our beliefs can serve as unassailable foundations for the rest of our beliefs is false. Holist criticisms undermine any given-ness of our purportedly basic beliefs. Given the constraints on knowledge, we could not know any particular fact unless we already knew a broader swath of background facts. We could not know that a spot is green unless we already knew that green is a color, that a spot which is green is not red, and so on.

> One couldn't form the concept of *being green*, and, by parity of reasoning, of the other colors, unless he already had them. (Sellars, "Empirical Knowledge," 120)

If knowing that this spot is green requires prior knowledge of a larger background theory, it becomes difficult to see how one could come to know anything at all. The holist, then, has a strong critical argument against the atomist, but creates what seems to be an even more intractable problem.

Summary

We have looked at two different kinds of responses to the color incompatibility problem. Carnap and Katz attempt to save atomism by exploring the logic of semantic entailments. Given first-order logic, there is no formal representation of the connections among 7S.11.1–7S.11.6. But we can extend our logic or our semantics so that there is a formal representation of those entailments.

In contrast, holists like Quine, Sellars, and Hempel give up the belief that there are elementary propositions. Quine, indeed, gives up on the idea that there are senses. Quine denies that there are any logical connections among 7S.11.1–7S.11.6; the connections are loose, at best causal connections.

One final note: the discussion of Wittgenstein's work in this section focuses on his early view, as represented in the *Tractatus*. Interestingly, Wittgenstein's later work is largely holistic. In *Philosophical Investigations*, he emphasizes connectedness. In even later work, Wittgenstein is explicitly holistic.

> "I set the brake up by connecting up rod and lever."—Yes, given the whole of the rest of the mechanism. Only in conjunction with that is it a brake-lever, and separated from its support it is not even a lever; it may be anything, or nothing. (Wittgenstein, *Philosophical Investigations*, section 6)

> When we first begin to *believe* anything, what we believe is not a single proposition, it is a whole system of propositions. (Light dawns gradually over the whole.) (Wittgenstein, *On Certainty*, section 141)

These topics are far too broad to be considered in proper depth here. We have reached the edge of logic and breached the barrier to the philosophy of language.

For Further Research and Writing

1. The logical empiricists were epistemic foundationalists, seeking to explain all of human knowledge on the basis of some secure, fundamental beliefs. Some critics of foundationalism, inspired by Quinean holism, defend coherentism in epistemology. Compare the two kinds of epistemologies. Sosa, Sellars, Ayer, and Quine would all be good readings.
2. In "Two Dogmas of Empiricism," Quine argues against the logical empiricist's reductionism. Evaluate Wittgenstein's project in light of Quine's criticisms. See Melchert for a good discussion of the *Tractatus*'s project, as well as Ayer.
3. Do meaning postulates solve the color incompatibility problem? See Carnap's "Meaning Postulates," in *Meaning and Necessity*, as well as Quine's response in "Two Dogmas of Empiricism."
4. What are semantic markers? How do they attempt to solve the color incompatibility problem? In addition to the discussion in Katz's "Problem," see Katz's *Semantic Theory*.
5. How does the color incompatibility problem shift Wittgenstein away from his original project? Work through his "Some Remarks on Logical Form." See Allaire and/or Austin as well.
6. What is the logical form of a sentence? Are there solutions, other than Carnap's, to the color incompatibility problem that rely on logical form? See the Pietroski article.

Suggested Readings

Allaire, Edwin. "Tractatus 6.3751." *Analysis* 19, no. 5 (1959): 100–105, 1959. A short and discussion of the color incompatibility problem, which connects Wittgenstein's later work with the *Tractatus*.

Austin, James. "Wittgenstein's Solutions to the Color Exclusion Problem." *Philosophy and Phenomenological Research* 41, no. 1/2 (1980): 142–149. A very good discussion, with excellent references.

Ayer, A. J. *Language, Truth, and Logic*. New York: Dover, 1952. An excellent discussion of the work of logical empiricism, following Wittgenstein's *Tractatus*.

Carnap, Rudolf. *Meaning and Necessity: A Study in Semantics and Modal Logic*. Chicago: University of Chicago Press, 1956 (Midway Reprint, 1988). See especially the paper on meaning postulates, at pp. 222ff.

Chisholm, Roderick. "The Myth of the Given." In *Epistemology: A Foundation*, edited by Ernest Sosa and Jaegwon Kim, 107–119. Malden, MA: Blackwell, 2000.

Chisholm, Roderick. "The Problem of the Criterion." In *Philosophy for the 21st Century*, edited by Steven M. Cahn, 152–160. New York: Oxford University Press, 2003.

Hempel, Carl. "Empiricist Criteria of Cognitive Significance: Problems and Changes." In *The Philosophy of Language*, 5th ed., edited by A. P. Martinich, 50–61. Oxford: Oxford University Press, 2008.

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