Box 14.2 Resistances of blood vessels in the arterial and capillary systems

From equations 14.3 and 14.4, resistance (R) to flow through a long straight tube is inversely proportional to the fourth power of the radius, r^4 and directly proportional to both the length of the tube, l, and the viscosity of the fluid, η, flowing through it. So:

$$R = \frac{8\eta l}{\pi r^4}$$

Multiplying by π produces: $R = \frac{8\eta l \pi}{\pi^2 r^4}$

The area, A, of a circle, is πr^2. Hence, $\pi^2 r^4 = A^2$

Thus, for tubes of the same length and fluid of the same viscosity flowing through them, $8\eta l\pi$ in the equation above can be represented by a constant, k and the resistance (R) to flow through one such tube will then be:

$$R = \frac{k}{A^2} \qquad \text{(a)}$$

Consider four identical tubes in parallel. From equation 14.5, the reciprocal of the total resistance of the four tubes (R_{tot}) will be:

$$\frac{1}{R_{tot}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{4}{R} \qquad \text{(b)}$$

Therefore, according to equation (a)

$$R_{tot} = \frac{R}{4} = \frac{k}{4A^2} \qquad \text{(c)}$$

More generally, the total resistance of n parallel identical tubes is:

$$R_{tot} = \frac{R}{n} = \frac{k}{nA^2} \qquad \text{(d)}$$

Now consider a single wide tube (such as a large artery) of cross sectional area A_L and four smaller tubes in parallel (such as small arteries) each with a cross sectional area of $A_s = A_L/4$ such that their total cross sectional area is equal to that of the large tube ($4\,A_S = A_L$). According to equation (a), the resistance of the large tube (R_L) is:

$$R_L = \frac{k}{A_L^2} \qquad \text{(e)}$$

According to equation (d), the total resistance of the four smaller tubes, R_{Stot}, is:

$$R_{Stot} = \frac{k}{4A_S^2} = \frac{k}{4\left(\frac{A_L}{4}\right)^2} = \frac{4k}{A_L^2} = 4R_L \qquad \text{(f)}$$

In other words, the total resistance of four small tubes, which have the same total cross sectional area as the large tube, equals four times the resistance of the large tube. So, what do these relationships tell us? Because of the large influence of the radius (r^4) on resistance, the total resistance of four smaller tubes in parallel is four times that of a larger tube of equal cross sectional area. Using a similar approach, we can calculate the total resistance of eight small tubes ($n = 8$), each with a quarter of the cross sectional area of the larger tubes ($A_S = A_L/4$), such that their total cross sectional area is twice that of the large tube ($8\,A_S = 2\,A_L$) According to equation (d):

$$R_{Stot} = \frac{k}{8A_S^2} = \frac{k}{8\left(\frac{A_L}{4}\right)^2} = \frac{2k}{A_L^2} = 2R_L \qquad \text{(g)}$$

This means that the resistance of the eight small parallel tubes is still twice that of the large tube, even though their total cross sectional area is twice that of the large tube. This is analogous to the comparison of the smallest arteries and arterioles with the large arteries.

Note that when there are 16 of the small tubes in parallel with a total cross-sectional area corresponding to four times that of the larger tube ($16\,A_S = 4\,A_L$), their total resistance will be the same as that of the large tube:

$$R_{Stot} = \frac{k}{16A_S^2} = \frac{k}{16\left(\frac{A_L}{4}\right)^2} = \frac{k}{A_L^2} = R_L \qquad \text{(h)}$$

Therefore, only when the total cross sectional area of the small tubes exceeds that of the large tube by more than four times, will their total resistance be lower than that of the large tube. This is analogous to the situation with respect to the arterioles and capillaries, as indicated in Figure 14.21. The radii (diameters/2) of different blood vessels in humans are given in Figure 14.6.

Further reading

Berne, RM and Levy, MN (Eds). 1998. Physiology 4th edn. Mosby, St Louis.