The chemist's toolkit 6 Measures of concentration

Let W be the solvent (such as water) and S the solute. The **molar concentration** (informally: 'molarity'), c_S or [S], is the amount of solute molecules (in moles) divided by the volume, V, of the solution:

$$\text{Molar concentration:} \quad c_S = \frac{n_S}{V} \text{ or } [S] = \frac{n_S}{V}$$

It is commonly reported in moles per cubic decimetre ($mol\,dm^{-3}$) or, equivalently, in moles per litre ($mol\,L^{-1}$). It is convenient to define its 'standard' value as $c^{\ominus} = 1\ mol\,dm^{-3}$.

The **molality,** b_S, of a solute is the amount of solute species (in moles) in a solution divided by the total mass of the solvent (in kilograms), m_W:

$$\text{Molality:} \quad b_S = \frac{n_S}{m_W}$$

Both the molality and mole fraction are independent of temperature; in contrast, the molar concentration is not. It is convenient to define the 'standard' value of the molality as $b^{\ominus} = 1\ mol\,kg^{-1}$.

1. **The relation between molality and mole fraction**

   Consider a solution with one solute and having a total amount n of molecules. If the mole fraction of the solute is x_S, the amount of solute molecules is $n_S = x_S n$. The mole fraction of solvent molecules is $x_W = 1 - x_S$, and so the amount of solvent molecules is $n_W = x_W n = (1 - x_S)n$. The mass of solvent, of molar mass M_W, present is $m_W = n_W M_W = (1 - x_S)nM_W$. The molality of the solute is therefore

$$b_S = \frac{n_S}{m_W} = \frac{x_S n}{(1-x_S)nM_W} = \frac{x_S}{(1-x_S)M_W}$$

   The inverse of this relation, the mole fraction in terms of the molality, is

$$x_S = \frac{b_S M_W}{1 + b_S M_W}$$

2. **The relation between molality and molar concentration**

   The total mass of a volume V of *solution* (not solvent) of mass density ρ is $m = \rho V$. The amount of solute molecules in this volume is $n_S = c_S V$. The mass of solute present is $m_S = n_S M_S = c_S V M_S$. The mass of solvent present is therefore $m_W = m - m_S = \rho V - c_S V M_S = (\rho - c_S M_S)V$. The molality is therefore

$$b_S = \frac{n_S}{m_W} = \frac{c_S V}{(\rho - c_S M_S)V} = \frac{c_S}{\rho - c_S M_S}$$

   The inverse of this relation, the molar concentration in terms of the molality, is

$$c_S = \frac{b_S \rho}{1 + b_S M_S}$$

3. **The relation between molar concentration and mole fraction**

   By inserting the expression for b_S in terms of x_S into the expression for c_S, the molar concentration of S in terms of its mole fraction is

$$c_S = \frac{x_S \rho}{x_W M_W + x_S M_S}$$

   with $x_W = 1 - x_S$. For a dilute solution in the sense that $x_S M_S << x_W M_W$,

$$c_S \approx \left(\frac{\rho}{x_W M_W}\right) x_S$$

   If, moreover, $x_S << 1$, so $x_W \approx 1$, then

$$c_S \approx \left(\frac{\rho}{M_W}\right) x_S$$