Groundwater hydraulics test - Answers

Exercise 1

a At the right hand side of the figure: the hydraulic gradient i along the line W equals 0.001; continuity: $|Q_0| = |Q_W|$; $Q_W = qA = qWD = -KiWD$ = –628.32 m^3 day^{-1} $\Rightarrow$ W = 1257 m

b $h_{tot} = h_r + h_x = h_R + \frac{Q_0}{2\pi T}\ln\frac{r}{R} + ir + C \Rightarrow \frac{dh_{tot}}{dr} = \frac{Q_0}{2\pi T}\left(\frac{1}{r}\right) + i$

$\frac{dh_{tot}}{dr} = 0 \Rightarrow$ (x,y) = (–200 m, 0 m)

Exercise 2

Pythagoras: $x^2 + 300^2 = 493.19^2 \Rightarrow x = 391.45$ m

r of the image recharge well = $\sqrt{(391.45 + 2 \times 101.43)^2 + 300^2} = 665.74$ m

$$h - h_\mathrm{R} = \frac{Q_0}{2\pi KD} \ln \frac{r}{R} ; \frac{Q_0}{2\pi KD} = \frac{314.16}{2\pi \times 10 \times 50} = 0.1$$

$h - h_\mathrm{R}$ in the midpoint of the nature reserve due to the pumping well =

$$0.1 \ln \frac{r}{R} = 0.1 \ln \frac{493.19}{2000} = -0.14 \text{ m}$$

$h - h_\mathrm{R}$ in the midpoint of the nature reserve due to the image recharge well =

$$-0.1 \ln \frac{r}{R} = -0.1 \ln \frac{665.74}{2000} = +0.11 \text{ m}$$

$h - h_\mathrm{R}$ in the midpoint of the nature reserve = –0.14 + 0.11 = –0.03 m $\Rightarrow$

the lowering of the hydraulic head $h_\mathrm{R} - h$ = 3 cm

Exercise 3:

$$0 \le x \le 20 : h = C_1 x + C_2$$

$$20 \le x \le \infty : h = h_a + C_1 e^{\frac{x}{\lambda}} + C_2 e^{\frac{-x}{\lambda}} ; \lambda = \sqrt{KDc} = 200 \text{ m}$$

$$x = 0 \Rightarrow h_0 = C_2 = 28.2 ; x = 20 \Rightarrow h_{20} = 20C_1 + 28.2 \Rightarrow C_1 = \frac{h_{20} - 28.2}{20} \Rightarrow$$

$$0 \le x \le 20 : h = \frac{h_{20} - 28.2}{20} x + 28.2$$

$$x = 20 \Rightarrow h_{20} = 26 + C_1 e^{\frac{20}{200}} + C_2 e^{\frac{-20}{200}}$$

$$x = \infty \Rightarrow 26 = 26 + C_1 e^{\infty} + C_2 e^{-\infty} \Rightarrow C_1 = 0 \Rightarrow$$

$$h_{20} = 26 + C_2 e^{\frac{-20}{200}} \Rightarrow C_2 = \frac{h_{20} - 26}{e^{\frac{-20}{200}}} \Rightarrow$$

$$20 \le x \le \infty : \ h = 26 + \frac{h_{20} - 26}{e^{\frac{-20}{200}}} e^{\frac{-x}{200}}$$

Continuity: $Q'_{0 \le x \le 20} = Q'_{20 \le x \le \infty} \Rightarrow \left(\frac{dh}{dx}\right)_{0 \le x \le 20} = \left(\frac{dh}{dx}\right)_{20 \le x \le \infty}$ at $x = 20 \Rightarrow$

$$\frac{h_{20} - 28.2}{20} = \frac{-1}{200} \times \frac{h_{20} - 26}{e^{\frac{-20}{200}}} e^{\frac{-x}{200}} = \frac{26 - h_{20}}{200} \Rightarrow h_{20} = 28 \text{ m} \Rightarrow$$

for $0 \le x \le 20 : h = -0.01x + 28.2$, and for $20 \le x \le \infty : h = 26 + 2e^{0.1} e^{\frac{-x}{200}}$

'Horizontal method':

$$Q'_z = Q'_{x=0} = -KD\left(\frac{dh}{dx}\right)_{x=0} = -500 \times -0.01 = 5 \text{ m}^2 \text{ day}^{-1}, \text{ or}$$

$$Q'_z = Q'_{x=20} = -KD\left(\frac{dh}{dx}\right)_{x=20} = -500 \times \left(\frac{-1}{200} \times 2 e^{0.1} e^{\frac{-x}{200}}\right)_{x=20} = 5 \text{ m}^2 \text{ day}^{-1}$$

$$Q'_{x=64.63} = -KD\left(\frac{dh}{dx}\right)_{x=64.63} = 4 \text{ m}^2 \text{ day}^{-1}; \ Q'_{x=20} - Q'_{x=64.63} = 1 \text{ m}^2 \text{ day}^{-1}$$

'Vertical method':

$$q_z = -k\frac{h_a - h}{z_a - z} = -k\frac{h_a - h}{d} = \frac{h - h_a}{c} = \frac{2e^{0.1}e^{\frac{-x}{200}}}{80};\ Q_z' = \int_{20}^{64.63} q_z\ dx = \int_{20}^{64.63} \frac{2}{80}e^{0.1}e^{\frac{-x}{200}}\ dx =$$

$$\left[-200\times\frac{2}{80}e^{0.1}\,e^{\frac{-x}{200}}\right]_{20}^{64/63} = -5e^{0.1}\left(e^{\frac{-64.63}{200}} - e^{\frac{-20}{200}}\right) = 1\ \mathrm{m}^2\ \mathrm{day}^{-1}$$