Derivation of the STORFLO simulation equation (after Kirkby 1975)

$$Q_t = Q_0 e^{\frac{S_t}{m}} \tag{5.80}$$

$$\ln Q_t = \ln\left(Q_0 e^{\frac{S_t}{m}}\right) \quad \Rightarrow \quad \ln Q_t = \ln Q_0 + \ln\left(e^{\frac{S_t}{m}}\right) \quad \Rightarrow \quad \ln Q_t = \ln Q_0 + \frac{S_t}{m} \quad \Rightarrow$$

$$S_t = m \ln Q_t - m \ln Q_0 * \quad \Rightarrow \quad \frac{dS_t}{dQ} = \frac{m}{Q_t} \quad \Rightarrow \quad dS_t = \frac{m}{Q_t} dQ$$

$$\frac{dS_t}{dt} = I - Q_t \tag{5.81}$$

$$\Rightarrow \quad dS_t = (I - Q_t)\, dt$$

Combining both expressions for dS_t yields

$$(I - Q_t)\, dt = \frac{m}{Q_t} dQ \quad \Rightarrow \quad -(Q_t - I)\, dt = \frac{m}{Q_t} dQ \quad \Rightarrow \quad -\frac{dt}{m} = \frac{dQ}{Q_t(Q_t - I)} \quad \Rightarrow$$

$$-\frac{dt}{m} = \frac{dQ}{I} \frac{I}{Q_t(Q_t - I)} \quad \Rightarrow \quad -\frac{dt}{m} = \frac{dQ}{I} \frac{Q_t - (Q_t - I)}{Q_t(Q_t - I)} \quad \Rightarrow$$

$$-\frac{dt}{m} = \frac{dQ}{I}\left(\frac{Q_t}{Q_t(Q_t - I)} - \frac{Q_t - I}{Q_t(Q_t - I)}\right) \quad \Rightarrow \quad -\frac{dt}{m} = \frac{dQ}{I}\left(\frac{1}{Q_t - I} - \frac{1}{Q_t}\right) \quad \Rightarrow$$

$$-\frac{I}{m} dt = \left(\frac{1}{Q_t - I} - \frac{1}{Q_t}\right) dQ \quad \Rightarrow \quad \int -\frac{I}{m}\, dt = \int \left(\frac{1}{Q_t - I} - \frac{1}{Q_t}\right) dQ \quad \Rightarrow$$

$$-\frac{I}{m} t = \ln(Q_t - I) - \ln Q_t + C \quad \Rightarrow \quad -\frac{I}{m} t = \ln\left(\frac{Q_t - I}{Q_t}\right) + C \quad \Rightarrow \quad \ln\left(\frac{Q_t - I}{Q_t}\right) = \frac{-It}{m} - C$$

$$\Rightarrow \quad \frac{Q_t - I}{Q_t} = e^{\frac{-It}{m} - C} \quad \Rightarrow \quad \frac{Q_t - I}{Q_t} = e^{\frac{-It}{m}} e^{-C} \quad \Rightarrow \quad \frac{Q_t - I}{Q_t} = A e^{\frac{-It}{m}} \quad \Rightarrow \quad Q_t - I = Q_t A e^{\frac{-It}{m}}$$

$$\Rightarrow \quad Q_t - Q_t A e^{\frac{-It}{m}} = I \quad \Rightarrow \quad Q_t\left(1 - Ae^{\frac{-It}{m}}\right) = I \quad \Rightarrow \quad Q_t = \frac{I}{1 - Ae^{\frac{-It}{m}}}$$

t = 0, then $Q_t = Q_0 \quad \Rightarrow \quad Q_0 = \frac{I}{1-A} \quad \Rightarrow \quad 1 - A = \frac{I}{Q_0} \quad \Rightarrow \quad A = 1 - \frac{I}{Q_0} \quad \Rightarrow$

$$Q_t = \frac{I}{1 - \left(1 - \frac{I}{Q_0}\right)e^{\frac{-It}{m}}} \quad \Rightarrow \quad Q_t = \frac{I}{1 - e^{\frac{-It}{m}} + \frac{I}{Q_0}e^{\frac{-It}{m}}}$$

For t = 1 and time step Δt = 1: $\quad Q_1 = \dfrac{I}{1 - e^{\frac{-I}{m}} + \frac{I}{Q_0}e^{\frac{-I}{m}}}$

For $t = t + \Delta t$ and time step Δt: $\quad Q_{t+\Delta t} = \dfrac{I}{1 - e^{\frac{-I}{m}} + \frac{I}{Q_t}e^{\frac{-I}{m}}}$ or

$$Q_{t+\Delta t} = \frac{I_{\Delta t}}{1 - e^{\frac{-I_{\Delta t}}{m}} + \frac{I_{\Delta t}}{Q_t}e^{\frac{-I_{\Delta t}}{m}}} \tag{5.82}$$

* In the STORFLO model (Kirkby *et al.* 1987):

$$S_{\text{reference}} = m \ln Q_{\text{sat}}$$

$$S_{\text{begin}} = S_{\text{reference}} - m \ln Q_0$$

Also:

$$I_{\Delta t} = \frac{Q_t + Q_{t+\Delta t}}{2} + OF_{\Delta t} + S_t \quad \text{with} \quad S_t = S_{\text{reference}} - m \ln Q_t \quad \Rightarrow$$

$$OF_{\Delta t} = I_{\Delta t} - \frac{Q_t + Q_{t+\Delta t}}{2} - \left(S_{\text{reference}} - m \ln Q_t\right)$$

References

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