2-Dimensional Analysis of Threshold Voltage Variation with Channel Length

Colin McAndrew
Freescale Semiconductor
Tempe, AZ

Abstract

The book (*Operation and Modeling of the MOS Transistor, 3rd Edition,* by Yannis Tsividis and Colin McAndrew, Oxford University Press, 2011) presents some results from analyses of the variation of threshold voltage V_T with channel length and drain bias (Sec. 5.4 on Charge Sharing and Sec. 5.5 on DIBL). A widely accepted result, based on 2-dimensional analysis of charge, was not included as it is somewhat more complex than most results presented in the book, and as with most 2-dimensional analyses includes some fairly significant approximations. To provide more detail a simplified derivation of the result is presented here.

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Threshold Voltage Model

The analysis presented here is adapted from [1]. The reader is cautioned that several approximations are necessary to lead to a closed form solution; these approximations are justified only in that they lead to results that can be fitted to experimental data.

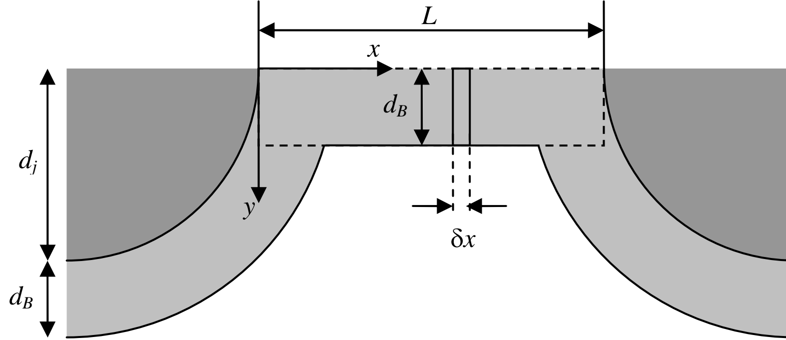

Fig. 1 Idealized depletion region under the gate.

Consider the depletion region under the gate in Fig. 5.7 to be approximated as a rectangular box of depth d_B that extends from the source to the drain (the situation is depicted in Fig. 1 above, as the dashed box). The 1-dimensional Poisson equation (A.3) needs to be extended to two dimensions to analyze this box,

$$(1) \qquad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} = -\frac{\rho(x, y)}{\varepsilon} = q\frac{N_A}{\varepsilon_s}$$

where $E_x = -\partial\psi/\partial x$ and $E_y = -\partial\psi/\partial y$ are the components of the electric field in the x and y directions, respectively. We consider operation in weak to moderate inversion therefore ignore the inversion charge in (1) and assume that the silicon charge is predominantly from the depletion region under the gate. Gauss's law can then be applied to a rectangular box of width δx and height d_B at position x along the channel (see solid rectangle near the center of the channel in Fig. 1), to give

$$(2) \qquad d_B\left[E_x(x) - E_x(x+\delta x)\right] + \delta x\left[E_y(0) - E_y(d_B)\right] = q\frac{d_B \delta x N_A}{\varepsilon_s}$$

where E_y is assumed to be independent of x and E_x is assumed to be independent of y. The latter assumption is clearly inaccurate; at the surface $E_x = -\partial\psi_s/\partial x$, but the potential at the bottom of the depletion region is ϕ_{MS} independent of position along the channel (see Fig. 2.5), therefore the electric field there in the x-direction there is zero. Nevertheless we will take the electric field along the vertical sides of the solid box in Fig. 1 to be constant and equal to its value at the surface, $-\partial\psi_s/\partial x$; adjustment of parameters to fit data will be assumed to compensate for inaccuracies. The electric field in the y-direction at the bottom of the depletion region is zero (see Fig. 2.14*c*), and at the surface it is, from (2.4.23) and (A.4),

$$(3) \qquad E_y(0) = \frac{C'_{ox}(V_{GB} - V_{FB} - \psi_s)}{\varepsilon_s}.$$

Introducing (3) in (2), and using (2.6.6) for d_B on the right-hand side of (2) (but not the left-hand side), and defining a characteristic or "gauge" length as in (5.5.2),

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(4) $$\lambda = \sqrt{\frac{\varepsilon_s t_{ox} d_B}{\varepsilon_{ox} \beta_3}} ,$$

gives

(5) $$\lambda^2 \frac{\partial^2 \psi_s}{\partial x^2} = \psi_s - \left(V_{GB} - V_{FB} - \frac{qN_A d_B}{C'_{ox}} \right) = \psi_s - \left(V_{GB} - V_{FB} - \gamma \sqrt{\psi_s} \right)$$

where we have used (2.6.6) and 2.4.26*a*). We have introduced the empirical fitting parameter β_3 in (4), which can be adjusted from its theoretical value of 1 to fit experimental data.

Now, in weak and moderate inversion for an ideal long channel transistor the surface potential is well approximated by ψ_{sa}, which is the solution of (2.5.4). ψ_{sa} is constant and independent of position along the channel, depending only on V_{GB}. Clearly for a long channel transistor in the center of the channel region ψ_s should be equal to ψ_{sa} and so be constant, hence $\partial \psi_s^2 / \partial x^2$ should be zero; this is consistent with (5) because, from (2.5.4), ψ_{sa} is the solution of

(6) $$V_{GB} - V_{FB} - \psi_s - \gamma \sqrt{\psi_s} = 0 .$$

At the ends of the channel, where the gradual channel approximation is not accurate and ψ_s does not equal ψ_{sa}, we will assume that, as per (6), $V_{GB} - V_{FB} - \gamma \sqrt{\psi_s} = \psi_{sa}$, and this then gives

(7) $$\frac{\partial^2 \psi_s}{\partial x^2} = \frac{\psi_s - \psi_{sa}}{\lambda^2} .$$

Note that this ignores the influence of the depletion regions from the source- and drain-bulk junctions at the ends of the channel, and part of the influence of ψ_s on d_B when ψ_s deviates from ψ_{sa} at the ends of the channel.

Because we are assuming operation in weak or moderate inversion, where there is negligible inversion charge, the surface potentials (measured with respect to the bulk) at the source and drain ends of the channel are $\psi_{s0} = V_{SB} + \phi_{bi}$ and $\psi_{sL} = V_{DB} + \phi_{bi}$ respectively, where ϕ_{bi} is the built-in potential of the junction (Sec. 1.5). Note that in the book we take the surface potential to be ψ_{sa} in weak inversion (Sec. 2.6.3), which is independent of V_{SB} and V_{DB} and constant along the channel, but this computation was based on a 1-dimensional analysis in the transverse direction and ignored details of the 2-dimensional nature of the electric field right at the ends of the channel. We are now assuming that right at the ends of the channel the surface potential is determined by a 1-dimensional analysis in the longitudinal direction, and are not taking the complete 2-dimensional nature of the problem into consideration. Solving (7) subject to the abovementioned boundary conditions for ψ_s at the ends of the channel gives

(8) $$\psi_s(x) = \psi_{sa} + \left(V_{SB} + \phi_{bi} - \psi_{sa} \right) \frac{\sinh\left(\frac{L-x}{\lambda} \right)}{\sinh\left(\frac{L}{\lambda} \right)} + \left(V_{DB} + \phi_{bi} - \psi_{sa} \right) \frac{\sinh\left(\frac{x}{\lambda} \right)}{\sinh\left(\frac{L}{\lambda} \right)} .$$

(As $\sinh(0) = 0$ it is apparent that the above expression reduces to $V_{SB} + \phi_{bi}$ for $x = 0$ and to $V_{DB} + \phi_{bi}$ for $x = L$. If L is very large with respect to λ then in the middle of the channel, for $x = L/2$, the terms in the ratios of the hyperbolic sines are both $e^{-L/(2\lambda)}$ which is very small, hence ψ_s approaches ψ_{sa}, as would be expected. If L is not large with respect to λ then ψ_s rises above ψ_{sa} in the middle of the channel as well as at the ends. This is shown in Fig. 2).

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We have seen (Sec. 2.6.2) that threshold voltage is the value of the gate voltage above which the surface potential is approximately pinned in strong inversion, to a value $\phi_0 = 2\phi_F + \Delta\phi$. The potential (8) is a minimum at approximately $x = L/2$ (at exactly this value for $V_{DS} = 0$), and the inversion charge density is also a minimum at this point. It is reasonable to consider that the *difference* in threshold voltage between a short and a long transistor is equivalent to the *difference* in ψ_s between a short and a long channel transistor at the onset of strong inversion such that the value of $\psi_s(x = L/2)$ from (8) is equal to its long channel value of ϕ_0. Setting $\psi_{sa} = \phi_0 + \Delta V_T$ in (8) and requiring $\psi_s(x = L/2) = \phi_0$ gives

$$\Delta V_T = -\frac{2(\phi_{bi} - \phi_0 + V_{SB}) + V_{DS}}{2\cosh\left(\frac{L}{2\lambda}\right) - 2} \tag{9}$$

This expression embodies the reduction in V_T both with increasing drain bias and with reducing channel length, and is the basis of the short channel threshold voltage model and the DIBL effect model in BSIM4. This is approximated in BSIM3 as

$$\Delta V_T = -\left(2(\phi_{bi} - \phi_0 + V_{SB}) + V_{DS}\right)\left(e^{-L/(2\lambda)} + 2e^{-L/\lambda}\right). \tag{10}$$

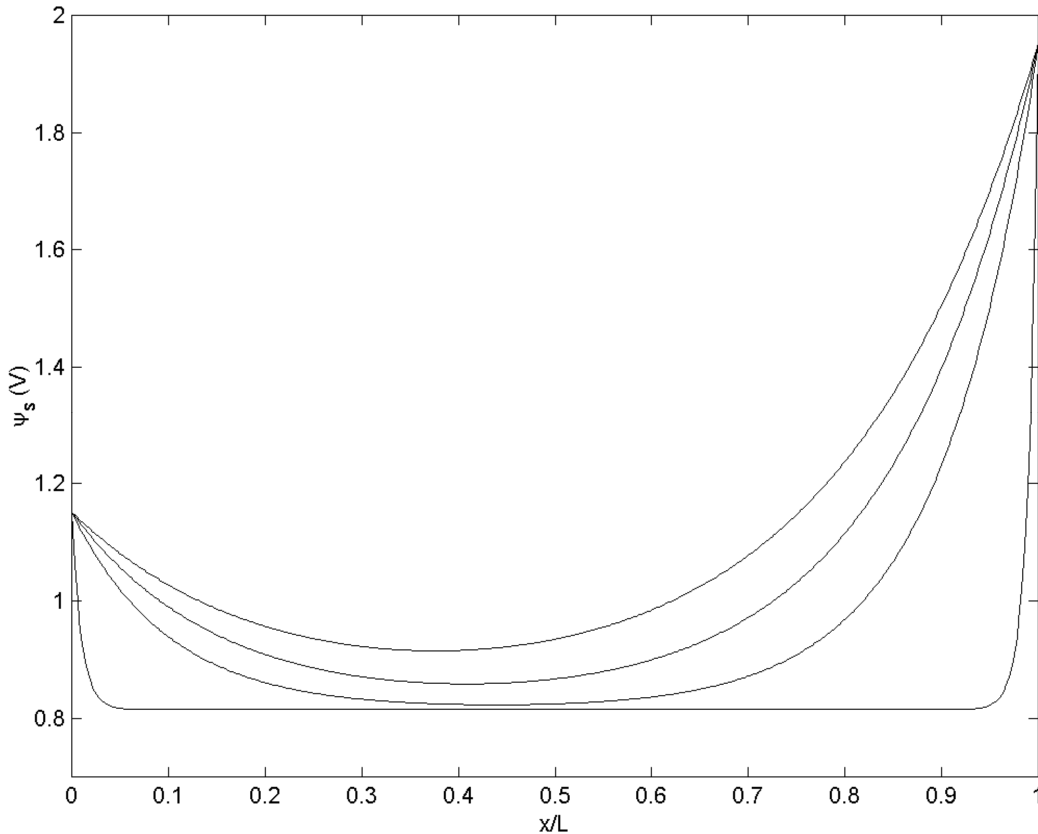

Fig. 2 Surface potential variation along the channel from (8) for $\lambda/L = 0.01$, 0.1, 0.15, and 0.2 from bottom to top. $t_{ox} = 2.0$ nm, $N_A = 10^{17}$ cm^{-3}, $V_{SB} = 0.2$ V, $V_{DB} = 1.0$ V, source and drain doping is 2×10^{19} cm^{-3}.

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References

[1] Z. H. Liu, C. Hu, J. H. Huang, T. Y. Chan, M.C . Jeng, P. K. Ko, and Y. C. Cheng, "Threshold voltage model for deep-submicrometer MOSFETs," *IEEE Tran. Electron Devices*, vol. 40, no. 1, pp. 86-95, Jan. 1993.

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