

# Complete and Consistent All-Region Body-Referenced Capacitance Model Based on Symmetric Linearization

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## Abstract

The book (*Operation and Modeling of the MOS Transistor, 3<sup>rd</sup> Edition*, by Yannis Tsividis and Colin McAndrew, Oxford University Press, 2011) presents a complete all-region  $I_{DS}$  model based on the bulk charge\* model (4.3.12) ( $Q_B = -\gamma C_{ox} \sqrt{\psi_s}$ ) and in Chap. 6 provides charge models, but does not present a complete and numerically stable charge model valid in all regions of operations; the details were considered too complex for the book and potentially distracting by providing too much mathematical detail which could detract from understanding of basic modeling approaches and physical device operation. Complete and consistent all-region dc and charge models, based on the symmetric linearization procedure, were provided in supplementary material on the books' website ([www.oup.com/us/tsividis\\_mcandrew](http://www.oup.com/us/tsividis_mcandrew)). With the advent of Verilog-A, dc and charge models (and noise, geometry scaling, and temperature models) are all that is required to implement a model for circuit simulation; the necessary derivatives, i.e. capacitances and conductances, are automatically generated from the Verilog-A code by Verilog-A compilers. However, it can sometimes be useful to have expressions for the capacitances themselves. Capacitances expressions for the symmetric linearization charge model are provided here.

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\* As in the book, we use the term “charge” rather than “charge per unit area” for simplicity. The fact that we are talking about charges *per unit area* will be clear from the primes used in the symbols for these quantities. The same applies to capacitances.

## Introduction

Complete charge expressions for the symmetric linearization all-region model [1] are derived in the supplementary material on the books' website ([www.oup.com/us/tsividis\\_mcandrew](http://www.oup.com/us/tsividis_mcandrew)) and are repeated below for convenience:

$$(1) \quad Q_G = LWC'_{ox} \left( V_{GB} - V_{FB} - \psi_{sm} + \frac{\Delta\psi_s^2}{12H} \right) - Q_o$$

$$(2) \quad Q_B = -\text{sgn}(\psi_{sm}) LWC'_{ox} \left[ \gamma \sqrt{\psi_{am}} - \frac{(\alpha_m - 1)\Delta\psi_s^2}{12H} \right]$$

$$(3) \quad Q_I = Q_S + Q_D = -LWC'_{ox} \left( \psi_{im} + \frac{\alpha_m \Delta\psi_s^2}{12H} \right)$$

$$(4) \quad Q_S = -\frac{LWC'_{ox}}{2} \left[ \psi_{im} + \frac{\alpha_m \Delta\psi_s}{6} \left( 1 + \frac{\Delta\psi_s}{2H} - \frac{\Delta\psi_s^2}{20H^2} \right) \right]$$

$$(5) \quad Q_D = -\frac{LWC'_{ox}}{2} \left[ \psi_{im} - \frac{\alpha_m \Delta\psi_s}{6} \left( 1 - \frac{\Delta\psi_s}{2H} - \frac{\Delta\psi_s^2}{20H^2} \right) \right]$$

where the midpoint surface potential is

$$(6) \quad \psi_{sm} = \frac{\psi_{s0} + \psi_{sL}}{2},$$

the potential that characterizes the normalized non-inversion silicon charge per unit area at the midpoint surface potential is

$$(7) \quad \psi_{am} = \phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t,$$

the potential that characterizes the normalized inversion layer charge per unit area at the midpoint surface potential is

$$(8) \quad \psi_{im} = V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\psi_{am}},$$

(which decreases approximately exponentially with decreasing gate bias as the transistor drops out of strong inversion) the difference in surface potential between the drain and source is

$$(9) \quad \Delta\psi_s = \psi_{sL} - \psi_{s0},$$

the bulk charge linearization factor is

$$(10) \quad \alpha_m = 1 + \frac{\gamma}{2\sqrt{\psi_{am}}} \left( 1 - e^{-\psi_{sm}/\phi_t} \right),$$

and

$$(11) \quad H = \phi_t + \frac{\psi_{im}}{\alpha_m}.$$

All other symbols have the same meaning as in the book. These expressions are valid for all regions of operation, from accumulation through strong inversion.

For a specific bias condition  $\psi_{s0} = \psi_{s0}(V_{GB}, V_{SB})$  and  $\psi_{sL} = \psi_{sL}(V_{GB}, V_{DB})$ , and several quantities above also depend on  $\psi_{s0}$  and  $\psi_{sL}$ , so the capacitance expressions will involve the derivative of these surface potentials with respect to the applied biases. One quantity that appears often in the derivation of the capacitances is

$$(12) \quad \frac{\partial H}{\partial \psi_{sm}} = - \left[ 1 + \frac{\psi_{im}}{\alpha_m^2 \sqrt{\psi_{am}}} \left( \frac{\gamma e^{-\psi_{sm}/\phi_t}}{2\phi_t} - \frac{(\alpha_m - 1)^2}{\gamma} \right) \right]$$

and for compactness of notation the expressions below will be written in terms of that quantity.

Using  $C_{ox} = WLC'_{ox}$  we have, for  $C_{mf} = -\partial Q_m / \partial V_f$ ,

$$(13) \quad C_{gs} = \frac{C_{ox}}{2} \left( 1 + \frac{\Delta \psi_s}{3H} + \frac{\Delta \psi_s^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right) \frac{\partial \psi_{s0}}{\partial V_{SB}}$$

$$(14) \quad C_{gd} = \frac{C_{ox}}{2} \left( 1 - \frac{\Delta \psi_s}{3H} + \frac{\Delta \psi_s^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right) \frac{\partial \psi_{sL}}{\partial V_{DB}}$$

$$(15) \quad C_{gb} = C_{ox} \left[ 1 + \frac{\Delta \psi_s}{6H} \left( \frac{\partial \psi_{sL}}{\partial V_{GB}} + \frac{\partial \psi_{sL}}{\partial V_{DB}} - \frac{\partial \psi_{s0}}{\partial V_{GB}} - \frac{\partial \psi_{s0}}{\partial V_{SB}} \right) - \frac{1}{2} \left( 1 + \frac{\Delta \psi_s^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right) \left( \frac{\partial \psi_{sL}}{\partial V_{GB}} + \frac{\partial \psi_{sL}}{\partial V_{DB}} + \frac{\partial \psi_{s0}}{\partial V_{GB}} + \frac{\partial \psi_{s0}}{\partial V_{SB}} \right) \right]$$

$$(16) \quad C_{bs} = (\alpha_m - 1) C_{gs} - \left[ \frac{\gamma e^{-\psi_{sm}/\phi_t}}{2\phi_t \sqrt{\psi_{am}}} - \frac{(\alpha_m - 1)^2}{\gamma \sqrt{\psi_{am}}} \right] \frac{\Delta \psi_s^2}{24H^2} \frac{\partial \psi_{s0}}{\partial V_{SB}}$$

$$(17) \quad C_{bd} = (\alpha_m - 1) C_{gd} - \left[ \frac{\gamma e^{-\psi_{sm}/\phi_t}}{2\phi_t \sqrt{\psi_{am}}} - \frac{(\alpha_m - 1)^2}{\gamma \sqrt{\psi_{am}}} \right] \frac{\Delta \psi_s^2}{24H^2} \frac{\partial \psi_{sL}}{\partial V_{DB}}$$

$$(18) \quad C_{sd} = -\frac{C_{ox}}{4} \left\{ \alpha_m + \frac{\Delta \psi_s}{6} \left[ \frac{\gamma e^{-\psi_{sm}/\phi_t}}{2\phi_t \sqrt{\psi_{am}}} - \frac{(\alpha_m - 1)^2}{\gamma \sqrt{\psi_{am}}} \right] \left( \frac{\Delta \psi_s^2}{20H^2} + \frac{\Delta \psi_s}{2H} - 1 \right) \right. \\ \left. - \frac{\alpha_m \Delta \psi_s^2}{12H^2} \left( \frac{\Delta \psi_s}{5H} + 1 \right) \frac{\partial H}{\psi_{sm}} + \frac{\alpha_m}{3} \left( \frac{3\Delta \psi_s^2}{20H^2} - \frac{\Delta \psi_s}{H} - 1 \right) \right\} \frac{\partial \psi_{sL}}{\partial V_{DB}}$$

The 3 other capacitive current elements in Fig. 8.5 involve the quantities

$$(19) \quad C_m = C_{dg} - C_{gd}$$

$$(20) \quad C_{mb} = C_{db} - C_{bd}$$

$$(21) \quad C_{mx} = C_{bg} - C_{gb}$$

and these can be determined from the capacitances above with the 3 additional capacitances

$$(22) \quad C_{dg} = \frac{C_{ox}}{2} \left( \begin{aligned} & 1 - \frac{\Delta\psi_s^2}{12H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) + \frac{\alpha_m}{6} \left( \frac{3\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{H} - 1 \right) \left( \frac{\partial\psi_{sL}}{\partial V_{GB}} - \frac{\partial\psi_{s0}}{\partial V_{GB}} \right) \\ & - \frac{1}{2} \left\{ \alpha_m - \frac{\Delta\psi_s}{6} \left[ \frac{\gamma e^{-\psi_{sm}/\phi_t}}{2\phi_t \sqrt{\psi_{am}}} - \frac{(\alpha_m - 1)^2}{\gamma \sqrt{\psi_{am}}} \right] \left( \frac{\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{2H} - 1 \right) + \frac{\alpha_m \Delta\psi_s^2}{12H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) \frac{\partial H}{\partial \psi_{sm}} \right\} \left( \frac{\partial\psi_{sL}}{\partial V_{GB}} + \frac{\partial\psi_{s0}}{\partial V_{GB}} \right) \end{aligned} \right)$$

$$(23) \quad C_{db} = \frac{C_{ox}}{2} \left( \begin{aligned} & -1 + \frac{\Delta\psi_s^2}{12H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) - \frac{\alpha_m}{6} \left( \frac{3\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{H} - 1 \right) \left( \frac{\partial\psi_{sL}}{\partial V_{GB}} + \frac{\partial\psi_{sL}}{\partial V_{DB}} - \frac{\partial\psi_{s0}}{\partial V_{GB}} - \frac{\partial\psi_{s0}}{\partial V_{SB}} \right) \\ & + \frac{1}{2} \left\{ \alpha_m - \frac{\Delta\psi_s}{6} \left[ \frac{\gamma e^{-\psi_{sm}/\phi_t}}{2\phi_t \sqrt{\psi_{am}}} - \frac{(\alpha_m - 1)^2}{\gamma \sqrt{\psi_{am}}} \right] \left( \frac{\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{2H} - 1 \right) + \frac{\alpha_m \Delta\psi_s^2}{12H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) \frac{\partial H}{\partial \psi_{sm}} \right\} \\ & \quad \times \left( \frac{\partial\psi_{sL}}{\partial V_{GB}} + \frac{\partial\psi_{sL}}{\partial V_{DB}} + \frac{\partial\psi_{s0}}{\partial V_{GB}} + \frac{\partial\psi_{s0}}{\partial V_{SB}} \right) \end{aligned} \right)$$

$$(24) \quad C_{bg} = C_{ox} \left( \begin{aligned} & \frac{(\alpha_m - 1)\Delta\psi_s^2}{12\alpha_m H^2} - (\alpha_m - 1) \frac{\Delta\psi_s}{6H} \left( \frac{\partial\psi_{sL}}{\partial V_{GB}} - \frac{\partial\psi_{s0}}{\partial V_{GB}} \right) \\ & + \frac{1}{2} \left\{ (\alpha_m - 1) - \left[ \frac{\gamma e^{-\psi_{sm}/\phi_t}}{2\phi_t \sqrt{\psi_{am}}} - \frac{(\alpha_m - 1)^2}{\gamma \sqrt{\psi_{am}}} \right] \frac{\Delta\psi_s^2}{12H} + \frac{(\alpha_m - 1)\Delta\psi_s^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right\} \left( \frac{\partial\psi_{sL}}{\partial V_{GB}} + \frac{\partial\psi_{s0}}{\partial V_{GB}} \right) \end{aligned} \right)$$

### Approximations for Accumulation and Depletion Operation

In accumulation and depletion the inversion charge is negligible, and to a good approximation  $\psi_{s0} = \psi_{sL} = \psi_{sm} = \psi_{sa}$  where  $\psi_{sa}$  is given by (3.2.24), and these depend only on  $V_{GB}$ ; all derivatives of surface potential with respect to  $V_{SB}$  and  $V_{DB}$  are zero, and we also have  $\Delta\psi_s = 0$ . We therefore directly get, from the capacitance expressions above,

$$(25) \quad C_{gs} = C_{gd} = C_{bs} = C_{bd} = C_{sd} = 0.$$

The term involving  $V_{CB}$  in the surface potential equation (3.2.11), which represents the mobile electron concentration, can be ignored in accumulation and depletion, so we have

$$(26) \quad (V_{GB} - V_{FB} - \psi_s)^2 = \gamma^2 (\phi_t e^{-\psi_s/\phi_t} + \psi_s - \phi_t)$$

and implicitly differentiating this with respect to  $V_{GB}$ , and comparing to (7) and (10) gives

$$(27) \quad \frac{\partial\psi_s}{\partial V_{GB}} = \frac{V_{GB} - V_{FB} - \psi_s}{V_{GB} - V_{FB} - \psi_s + 0.5\gamma^2 (1 - e^{-\psi_s/\phi_t})} = \frac{1}{\alpha_m}.$$

Using this, in addition to the information about the terms that go to zero, in (22) and (23), gives

$$(28) \quad C_{dg} = C_{db} = 0.$$

Finally, from (15) and (24) we have, using (27),

$$(29) \quad C_{gb} = C_{bg} = C_{ox} \left( 1 - \frac{\partial\psi_s}{\partial V_{GB}} \right) = C_{ox} \left( 1 - \frac{1}{\alpha_m} \right)$$

therefore also  $C_m = C_{mb} = C_{mx} = 0$ .

It could be intuitively expected that only  $C_{gb}$  and  $C_{bg}$  should be nonzero. In depletion and accumulation there is no inversion charge, so all derivatives of source and drain charges should be zero, and the source and drain regions cannot “communicate” with the MOS transistor and so cannot influence its behavior, therefore all derivatives with respect source and drain voltages should be zero. The charge balance within a MOS transistor then is between the gate charge and the bulk charge, the latter of which consists of depletion charge due to the ionized acceptor atoms and the accumulation charge due to mobile holes. The gate and bulk charges balance, and they vary with  $\psi_s$ , which is a function of  $V_{GB}$ , so  $C_{gb}$  should be nonzero. From Fig. 2.12 we see that as  $V_{GB}$  becomes increasingly negative below the flatband voltage, the derivative  $\partial\psi_s/\partial V_{GB}$  starts at a positive value (which will be developed below) and monotonically decreases, asymptotically approaching zero for  $V_{GB} \rightarrow -\infty$ .  $C_{gb}$  therefore starts at a finite value less than  $C_{ox}$  at  $V_{GB} = V_{FB}$  and asymptotically increases toward  $C_{ox}$  as  $V_{GB}$  becomes more negative, as Fig. 2.25 shows. Accurate modeling of capacitances around flatband is not important for MOS transistors, which are generally operated at higher values of  $V_{GB}$ , but is critical for modeling of MOS devices that are used as voltage variable capacitors [2][3][4][5].

The final expression in (29) in fact holds for all gate biases in the absence of inversion charge, and using (10) in (29) gives the high-frequency capacitance behavior curve of Fig. 2.25.

For operation in depletion somewhat above flatband, the  $e^{-\psi_s/\phi_t}$  term, which relates to the accumulation component of charge, is negligible. Implicit differentiation of (2.5.4) gives  $\partial\psi_s/\partial V_{GB} = 1/(1 + 0.5\gamma/\sqrt{\psi_s})$  and substituting this in (29) gives

$$(30) \quad C_{gb} = C_{ox} \left( \frac{\gamma}{\gamma + 2\sqrt{\psi_{sa}}} \right) = C_{ox} \left( \frac{\gamma}{2\sqrt{\gamma^2/4 + V_{GB} - V_{FB}}} \right).$$

Note that (30) predicts that at  $V_{GB} = V_{FB}$  we should have  $C_{gb} = C_{ox}$ , which is inaccurate; the gate-bulk capacitance, as we will see shortly, is less than  $C_{ox}$  at the flatband condition. Equation (30) is therefore only a reasonable approximation for operation where the  $e^{-\psi_s/\phi_t}$  term in (3.2.11) (without the inversion charge component) is small compared to unity. Our analysis here in accumulation and depletion is based on the bulk charge model

$$(31) \quad Q_B' = -\text{sgn}(\psi_s) \sqrt{2q\epsilon_s N_A} \sqrt{\phi_t e^{-\psi_s/\phi_t} + \psi_s - \phi_t}$$

which is valid in all regions of operation, yet the analysis leading to the definition of  $\psi_{sa}$  in the book is based on the simplified bulk charge model (2.6.7). This main seem contradictory, but is not; we are using the definition (2.5.5) of  $\psi_{sa}$  only to simplify the expressions.

For operation at flatband L’Hôpital’s rule can be applied to (27), which results, after some manipulation, in

$$(32) \quad \left. \frac{\partial\psi_s}{\partial V_{GB}} \right|_{V_{GB}=V_{FB}} = \frac{1}{1 + \gamma/\sqrt{2\phi_t}}$$

and using this in (29) we have

$$(33) \quad C_{gb} \Big|_{V_{GB}=V_{FB}} = C_{ox} \left( \frac{1}{1 + \sqrt{2\phi_t}/\gamma} \right).$$

The gate-substrate capacitance is therefore less than  $C_{ox}$  at flatband, by an amount that depends on the gate dielectric thickness and substrate doping, which affect  $\gamma$ , and on the temperature, which affects  $\phi_t$ .

In accumulation, the exponential term in the right hand side of (26) dominates. Keeping only this term and using the resulting approximation in (29) gives

$$(34) \quad C_{gb} = C_{ox} \left( \frac{V_{GB} - V_{FB} - \psi_s}{V_{GB} - V_{FB} - \psi_s - 2\phi_t} \right) \approx C_{ox} \left( 1 + \frac{2\phi_t}{V_{GB} - V_{FB} - \psi_s} \right).$$

In accumulation  $V_{GB} - V_{FB} - \psi_s$  is negative, so as expected from our previous discussions  $C_{gb}$  will asymptotically increase towards the value  $C_{ox}$  as  $V_{GB}$  becomes more negative below flatband.

### Approximations for Strong Inversion Nonsaturation Operation

In strong inversion nonsaturation operation, the source and drain surface potentials are approximately pinned to  $\phi_0 + V_{SB}$  and  $\phi_0 + V_{DB}$ , respectively, and to first order do not depend on  $V_{GB}$ , so

$$(35) \quad \frac{\partial \psi_{s0}}{\partial V_{SB}} = \frac{\partial \psi_{sL}}{\partial V_{DB}} = 1,$$

$$(36) \quad \frac{\partial \psi_{s0}}{\partial V_{GB}} = \frac{\partial \psi_{sL}}{\partial V_{GB}} = 0,$$

$$(37) \quad \Delta \psi_s = V_{DS}.$$

Additionally, the terms in  $e^{-\psi_s/\phi_t}$ , which relate to the accumulation holes, are negligible, and  $\psi_{sm} \gg \phi_t$ . The quantities associated with the symmetric linearization charge sheet model therefore become, in terms of terminal voltages,

$$(38) \quad \psi_{sm} = \phi_0 + 0.5(V_{SB} + V_{DB}) = \phi_0 + V_{SB} + 0.5V_{DS},$$

$$(39) \quad \psi_{am} = \psi_{sm},$$

$$(40) \quad \psi_{im} = V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\psi_{sm}},$$

$$(41) \quad \alpha_m = 1 + \frac{\gamma}{2\sqrt{\psi_{sm}}},$$

with  $H$  still given by (11) but with  $\psi_{im}$  from (40), and the derivative of  $H$  with respect to  $\psi_{sm}$ , (12), is

$$(42) \quad \frac{\partial H}{\partial \psi_{sm}} = - \left( 1 - \frac{(\alpha_m - 1)\psi_{im}}{2\alpha_m^2 \psi_{sm}} \right).$$

Using these in the general capacitance expressions (13) through (24) gives

$$(43) \quad C_{gs} = \frac{C_{ox}}{2} \left( 1 + \frac{V_{DS}}{3H} + \frac{V_{DS}^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right)$$

$$(44) \quad C_{gd} = \frac{C_{ox}}{2} \left( 1 - \frac{V_{DS}}{3H} + \frac{V_{DS}^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right)$$

$$(45) \quad C_{gb} = -C_{ox} \frac{V_{DS}^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}}$$

$$(46) \quad C_{bs} = (\alpha_m - 1)C_{gs} + \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \frac{V_{DS}^2}{24H^2}$$

$$(47) \quad C_{bd} = (\alpha_m - 1)C_{gd} + \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \frac{V_{DS}^2}{24H^2}$$

$$(48) \quad C_{sd} = -\frac{C_{ox}}{4} \left( \alpha_m - \frac{V_{DS}}{6} \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \left( \frac{V_{DS}^2}{20H^2} + \frac{V_{DS}}{2H} - 1 \right) + \frac{\alpha_m}{6} \left( \frac{3V_{DS}^2}{20H^2} - \frac{V_{DS}}{H} - 1 \right) - \frac{\alpha_m V_{DS}^2}{12H^2} \left( \frac{V_{DS}}{5H} + 1 \right) \frac{\partial H}{\partial \psi_{sm}} \right)$$

$$(49) \quad C_{dg} = \frac{C_{ox}}{2} \left( 1 - \frac{V_{DS}^2}{12H^2} \left( \frac{V_{DS}}{5H} + 1 \right) \right)$$

$$(50) \quad C_{db} = \frac{C_{ox}}{2} \left( \alpha_m - 1 + \frac{V_{DS}^2}{12H^2} \left( \frac{V_{DS}}{5H} + 1 \right) + \frac{V_{DS}}{6} \left( \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \right) \left( \frac{V_{DS}^2}{20H^2} + \frac{V_{DS}}{2H} - 1 \right) + \frac{\alpha_m V_{DS}^2}{12H^2} \left( \frac{V_{DS}}{5H} + 1 \right) \frac{\partial H}{\partial \psi_{sm}} \right)$$

$$(51) \quad C_{bg} = C_{ox} \left( \frac{(\alpha_m - 1)V_{DS}^2}{12\alpha_m H^2} \right).$$

Although these expressions depend on  $H$  and  $\partial H/\partial \psi_{sm}$ , which in turn depend on  $V_{DS}$ ,  $V_{GB}$ , and  $V_{SB}$ , they give explicit formulas for the capacitances from the symmetric linearization charge sheet model in strong inversion nonsaturation.

In contrast to the general expressions for  $V_{DS} = 0$ , that we will derive below, which predict a small positive value for  $C_{gb}$  and  $C_{bg}$  under that bias condition, in deriving the strong inversion approximations we have ignored the small variation of surface potential with gate voltage, which is why (45) and (51) predict that  $C_{gb} = C_{bg} = 0$  for  $V_{DS} = 0$ . Careful inspection of Fig. 1 and Fig. 2, below, show the slight nonzero value at  $V_{DS} = 0$ . Similarly, (43) and (44) predict that at  $V_{DS} = 0$  we should have  $C_{gs} = C_{gd} = C_{ox}/2$ ; Fig. 1 shows that these capacitances are in fact slightly smaller than  $C_{ox}/2$ ; this is because the surface potentials are not exactly pinned to  $\phi_0 + V_{SB}$  and  $\phi_0 + V_{DB}$  in strong inversion.

### Approximations for Strong Inversion Saturation Operation

There is no abrupt transition at saturation, rather as  $V_{DS}$  increases the drain end of the channel gradually drops out of strong inversion, and will eventually enter weak inversion for sufficiently large drain bias. At this point, using the weak inversion analysis of Sec. 2.6.3, we have

$$(52) \quad \psi_{sL} = \psi_{sa}$$

where  $\psi_{sa}$  is given by (2.6.27),

$$(53) \quad \frac{\partial \psi_{sL}}{\partial V_{DB}} = 0,$$

$$(54) \quad \frac{\partial \psi_{sL}}{\partial V_{GB}} = 1 + \frac{\gamma}{2\sqrt{\psi_{sa}}}$$

and we will still have  $\psi_{s0} = \phi_0 + V_{CB}$ ,  $\partial \psi_{s0}/\partial V_{SB} = 1$ , and  $\partial \psi_{s0}/\partial V_{GB} = 0$ . The difference between the drain and source surface potentials is then

$$(55) \quad \Delta \psi_s = \psi_{sa} - \phi_0 - V_{SB},$$

the midpoint surface potential is

$$(56) \quad \psi_{sm} = \frac{\phi_0 + V_{SB} + \psi_{sa}}{2},$$

and (39) through (42) are still valid with  $\psi_{sm}$  from (56). Using these simplifications in (13) through (24) gives

$$(57) \quad C_{gs} = \frac{C_{ox}}{2} \left( 1 + \frac{\Delta\psi_s}{3H} + \frac{\Delta\psi_s^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right)$$

$$(58) \quad C_{gd} = 0$$

$$(59) \quad C_{gb} = C_{ox} \left( 1 + \frac{\Delta\psi_s}{12H} \frac{\gamma}{\sqrt{\psi_{sa}}} - \left( 1 + \frac{\Delta\psi_s^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right) \left( 1 + \frac{\gamma}{4\sqrt{\psi_{sa}}} \right) \right)$$

$$(60) \quad C_{bs} = (\alpha_m - 1)C_{gs} + \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \frac{\Delta\psi_s^2}{24H^2}$$

$$(61) \quad C_{bd} = 0$$

$$(62) \quad C_{sd} = 0$$

$$(63) \quad C_{dg} = \frac{C_{ox}}{2} \left\{ \begin{aligned} & 1 - \frac{\Delta\psi_s^2}{12H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) + \frac{\alpha_m}{6} \left( \frac{3\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{H} \right) \left( 1 + \frac{\gamma}{2\sqrt{\psi_{sa}}} \right) \\ & - \left[ \frac{2\alpha_m}{3} + \frac{\Delta\psi_s}{12} \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \left( \frac{\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{2H} - 1 \right) + \frac{\alpha_m \Delta\psi_s^2}{24H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) \frac{\partial H}{\partial \psi_{sm}} \right] \left( 1 + \frac{\gamma}{2\sqrt{\psi_{sa}}} \right) \end{aligned} \right\}$$

$$(64) \quad C_{db} = \frac{C_{ox}}{2} \left\{ \begin{aligned} & -1 + \frac{\Delta\psi_s^2}{12H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) - \frac{\alpha_m}{3} \left( \frac{3\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{H} \right) \left( 1 + \frac{\gamma}{4\sqrt{\psi_{sa}}} \right) \\ & + \left[ \frac{4\alpha_m}{3} + \frac{\Delta\psi_s}{6} \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \left( \frac{\Delta\psi_s^2}{20H^2} + \frac{\Delta\psi_s}{2H} - 1 \right) + \frac{\alpha_m \Delta\psi_s^2}{12H^2} \left( \frac{\Delta\psi_s}{5H} + 1 \right) \frac{\partial H}{\partial \psi_{sm}} \right] \left( 1 + \frac{\gamma}{4\sqrt{\psi_{sa}}} \right) \end{aligned} \right\}$$

$$(65) \quad C_{bg} = C_{ox} \left\{ \frac{(\alpha_m - 1)\Delta\psi_s^2}{12\alpha_m H^2} + \frac{1}{2} \left[ (\alpha_m - 1) \left( 1 - \frac{\Delta\psi_s}{3H} \right) + \frac{(\alpha_m - 1)^2}{\gamma\sqrt{\psi_{am}}} \frac{\Delta\psi_s^2}{12H} + \frac{(\alpha_m - 1)\Delta\psi_s^2}{12H^2} \frac{\partial H}{\partial \psi_{sm}} \right] \left( 1 + \frac{\gamma}{2\sqrt{\psi_{sa}}} \right) \right\}.$$

### Exact Capacitances for $V_{ds}=0$

If  $V_{DS} = 0$  we have  $\psi_{s0} = \psi_{sL} = \psi_{sm} = \psi_s$ , so  $\Delta\psi_s = 0$ , and  $\partial\psi_{s0}/\partial V_{GB} = \partial\psi_{sL}/\partial V_{GB} = \partial\psi_s/\partial V_{GB}$ . The total capacitance at the gate terminal  $C_{gg}$  is, summing (13), (14), and (15),

$$(66) \quad C_{gg} = C_{gs} + C_{gd} + C_{gb} = C_{ox} \left( 1 - \frac{\partial\psi_s}{\partial V_{GB}} \right).$$

This result is general and holds for  $V_{DS} = 0$  for all regions of operation. Implicit differentiation of the surface potential equation (3.2.11) gives (Prob. 3.18), without simplification for a specific region of operation,

$$(67) \quad \frac{\partial \psi_s}{\partial V_{GB}} = \frac{V_{GB} - V_{FB} - \psi_s}{V_{GB} - V_{FB} - \psi_s + 0.5\gamma^2 \left( 1 - e^{-\psi_s/\phi_t} + e^{-(2\phi_F + V_{CB})/\phi_t} (e^{\psi_s/\phi_t} - 1) \right)}.$$

As  $\psi_s$  becomes less the zero, i.e. for operation in accumulation, the  $e^{-\psi_s/\phi_t}$  term in the denominator of this expression becomes large and  $\partial \psi_s / \partial V_{GB}$  approaches zero. Similarly, for  $\psi_s \gg 2\phi_F + V_{CB}$ , i.e. for operation in strong inversion, the  $e^{\psi_s/\phi_t}$  term in the denominator of this expression becomes large and  $\partial \psi_s / \partial V_{GB}$  again approaches zero. Hence  $C_{gg}$  will asymptotically approach  $C_{ox}$  in both accumulation and strong inversion. Using the relations (66) and (67) gives the low-frequency behavior capacitance curve (solid line) of Fig. 2.25.

The capacitances (13) through (24) simplify to

$$(68) \quad C_{gs} = C_{gd} = \frac{C_{ox}}{2} \frac{\partial \psi_s}{\partial V_{CB}}$$

$$(69) \quad C_{gb} = C_{ox} \left( 1 - \left( \frac{\partial \psi_s}{\partial V_{GB}} + \frac{\partial \psi_s}{\partial V_{CB}} \right) \right)$$

$$(70) \quad C_{bs} = C_{bd} = (\alpha_m - 1) \frac{C_{ox}}{2} \frac{\partial \psi_s}{\partial V_{CB}}$$

$$(71) \quad C_{sd} = C_{ox} \left( -\frac{\alpha_m}{6} \frac{\partial \psi_s}{\partial V_{DB}} \right)$$

$$(72) \quad C_{dg} = \frac{C_{ox}}{2} \left( 1 - \alpha_m \frac{\partial \psi_s}{\partial V_{GB}} \right)$$

$$(73) \quad C_{db} = \frac{C_{ox}}{2} \left( \alpha_m \left( \frac{\partial \psi_s}{\partial V_{GB}} + \frac{\partial \psi_s}{\partial V_{CB}} \right) - 1 \right)$$

$$(74) \quad C_{bg} = C_{ox} \left( (\alpha_m - 1) \frac{\partial \psi_s}{\partial V_{GB}} \right)$$

$$(75) \quad C_{mx} = 2C_{mb} = -2C_m = C_{ox} \left[ \left( \alpha_m \frac{\partial \psi_s}{\partial V_{GB}} + \frac{\partial \psi_s}{\partial V_{CB}} \right) - 1 \right].$$

This last expression shows that the body-referenced all-region has a small non-reciprocity in capacitances at  $V_{DS} = 0$ .

In strong inversion  $\partial \psi_s / \partial V_{GB}$  is not exactly equal to zero, it has a small positive value, and  $\partial \psi_s / \partial V_{CB}$  is not exactly 1, it is slightly less. The magnitude of the capacitances in (75) therefore falls off rather slowly, approximately inversely with increasing gate voltage, in strong inversion. As the gate voltage decreases and operation transitions to weak inversion, depletion, and accumulation,  $\partial \psi_s / \partial V_{CB}$  approaches zero and from (27)  $\partial \psi_s / \partial V_{GB}$  approaches  $1/\alpha_m$ , therefore the amount of non-reciprocity approaches zero (it does so very rapidly as the gate voltage is reduced, because the inversion charge decreases approximately exponentially with gate voltage in weak inversion).

This nonreciprocal behavior, even at  $V_{DS} = 0$ , is a consequence of the charge sheet approximation [6]; TCAD simulation and the Pao-Sah model [7], which is the reference for all MOSFET models (but involves double integrals and is requires numerical solution, so is not suitable for analytic or circuit simulation models), satisfy reciprocity at  $V_{DS} = 0$ . The

amount of non-reciprocity is small, and has not proven to cause problems for circuit design. The above formula (75) quantifies the degree of non-reciprocity in the body-referenced all-region model based on symmetric linearization.

For the case of strong inversion where  $V_{DS} = 0$ , the source and drain surface potentials are approximately pinned to  $\phi_0 + V_{SB}$  and  $\phi_0 + V_{DB}$ , respectively, and to first order do not depend on  $V_{GB}$ , so (35) through (37) hold and we have

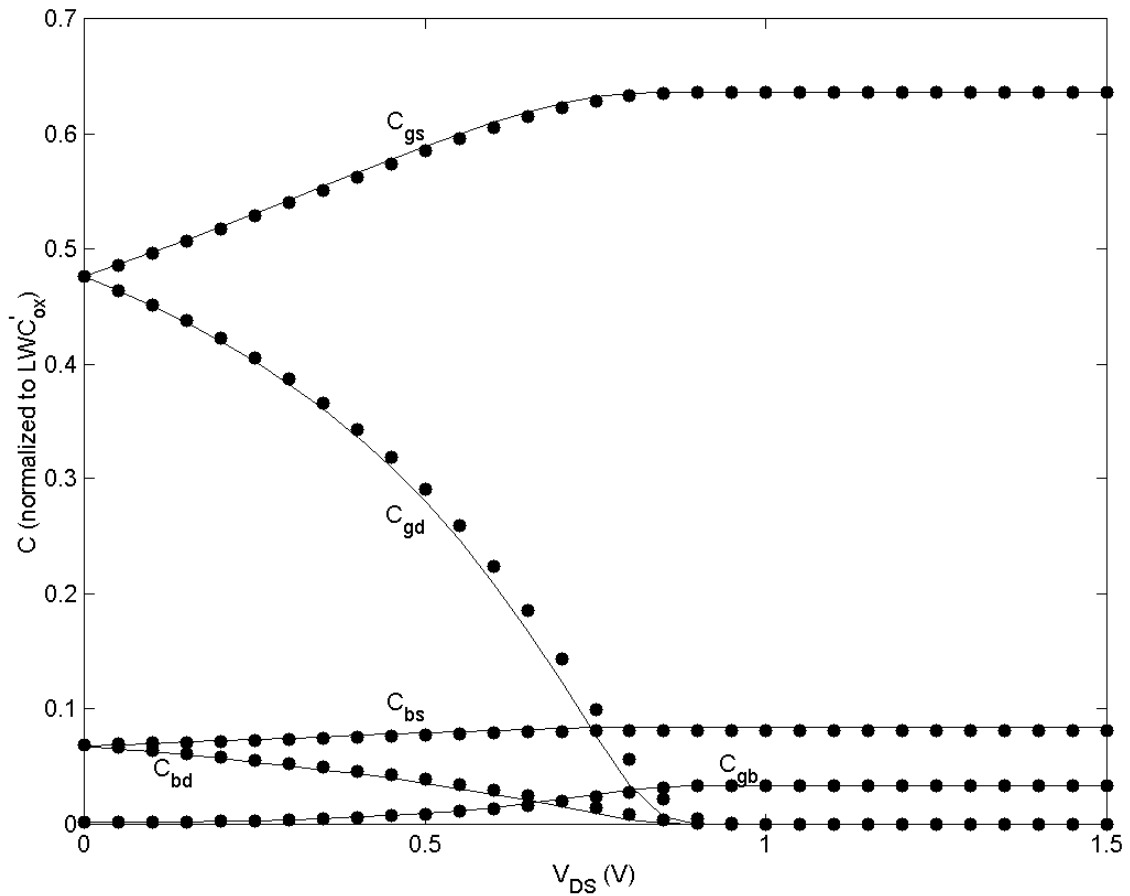
$$(76) \quad C_{gs} = C_{gd} = C_{dg} = \frac{C_{ox}}{2}$$

$$(77) \quad C_{gb} = C_{bg} = 0$$

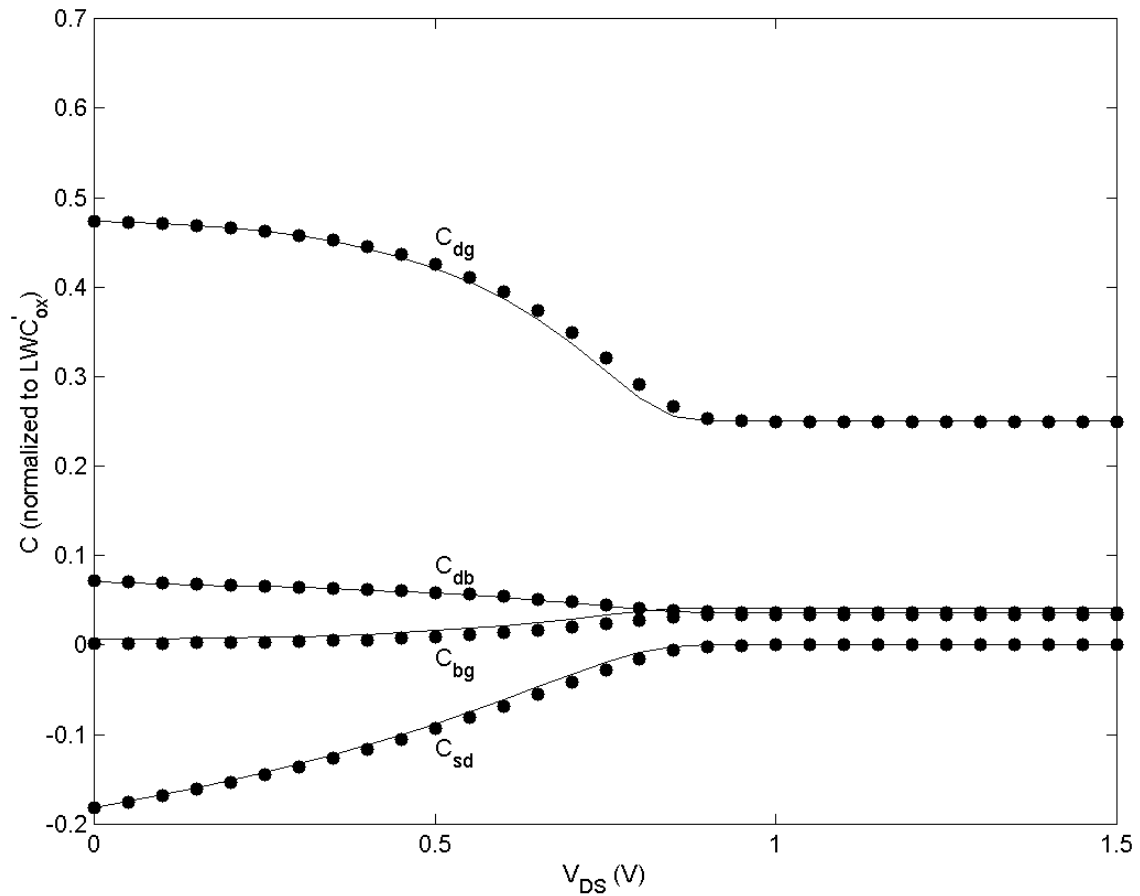
$$(78) \quad C_{bs} = C_{bd} = C_{db} = (\alpha_m - 1) \frac{C_{ox}}{2} = C_{ox} \frac{\gamma}{2\sqrt{\phi_0 + V_{CB}}}$$

$$(79) \quad C_{sd} = -\frac{\alpha_m}{6} C_{ox}$$

therefore also  $C_m = C_{mb} = C_{mx} = 0$ .



**Fig. 1** Capacitances for the medium frequency small-signal model of Chap 7, for the complete all-region model (lines) and the body-referenced all-region symmetric linearization model (symbols).  $W/L=10\mu\text{m}/10\mu\text{m}$ ,  $V_{FB}=-0.8$ ,  $N_A=5\times 10^{17}\text{cm}^{-3}$ ,  $t_{ox}=25\text{\AA}$ ,  $V_{SB}=0$ , and  $V_{GB}=1.4\text{V}$ . This is for operation in strong inversion; the complete all-region model is not valid in accumulation or the bottom of depletion.



**Fig. 2** Additional capacitances for the high frequency small-signal model of Chap 8, for the complete all-region model (lines) and the body-referenced all-region symmetric linearization model (symbols).  $W/L=10\mu\text{m}/10\mu\text{m}$ ,  $V_{FB}=-0.8$ ,  $N_A=5\times 10^{17}\text{cm}^{-3}$ ,  $t_{ox}=25\text{\AA}$ ,  $V_{SB}=0$ , and  $V_{GB}=1.4\text{V}$ . This is for operation in strong inversion; the complete all-region model is not valid in accumulation or the bottom of depletion.

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