

# Complete and Consistent All-Region Body-Referenced dc and Charge Models Based on Symmetric Linearization

Colin McAndrew  
Freescall Semiconductor  
Tempe, AZ

## Abstract

The book (*Operation and Modeling of the MOS Transistor, 3<sup>rd</sup> Edition*, by Yannis Tsividis and Colin McAndrew, Oxford University Press, 2011) presents a complete all-region  $I_{DS}$  model based on the bulk charge\* model (4.3.12) ( $Q_B = -\gamma C_{ox} \sqrt{\psi_s}$ ) and in Chap. 6 provides charge models, but does not present a complete and numerically stable charge model valid in all regions of operations (although this is addressed in Probs. 6.7 and 6.8). For detailed and intuitive understanding of device operation, for students and CMOS circuit designers, this is adequate. However, the bulk charge model (4.3.12) is not accurate for  $\psi_s$  less than several  $\phi_t$  (i.e. for  $V_{GB}$  from below to slightly above  $V_{FB}$ ), and for the purposes of developing high quality “industrial strength” models for circuit simulation it is desirable to have a complete, consistent, and numerically stable  $I_{DS}$  and charge model formulation that is valid in all regions of operation. Such a model can be complex, and can obfuscate the important and essential mathematical details of MOS transistor modeling and operation for students and for non-compact-modeling professionals. For this reason such a model was not included in the main text of the book. However, for researchers interested in the details of an “industrial strength” MOS transistor modeling a complete and consistent all-region model for  $I_{DS}$  and charges is essential; it is provided here. At present the only known tractable solution, which is both simple and elegant, is based on the symmetric linearization procedure of [1], which underlies the formulation of the PSP model [2]. The derivation and model presented here apply this procedure to a bulk charge model valid in all regions of operation, not just for  $V_{GB}$  somewhat above flatband.

---

\* As in the book, we use the term “charge” rather than “charge per unit area” for simplicity. The fact that we are talking about charges *per unit area* will be clear from the primes used in the symbols for these quantities. The same applies to capacitances.

## Introduction

Here we consider operation of an  $n$ -channel MOS transistor with a uniformly doped  $p$ -type substrate with doping acceptor density  $N_A$ . We assume that the gradual channel approximation (Sec. 4.1) is valid. With no further simplification, the semiconductor charge is (3.2.8)

$$(1) \quad Q_C' = -\text{sgn}(\psi_s) \sqrt{2q\epsilon_s N_A} \sqrt{\phi_t e^{-\psi_s/\phi_t} + \psi_s - \phi_t + e^{-(2\phi_F + V_{CB})/\phi_t} (\phi_t e^{\psi_s/\phi_t} - \psi_s - \phi_t)}$$

where we note (Appendix E) that this differs from the classic result (Ref. [3], given as (I.5) in Appendix I of [4]) in that it is based on requiring charge neutrality at flat-band. This overcomes the serious numerical issue, that arises in computer simulation based on the classic result, of the argument of the second square root term in (1) becoming negative for  $\psi_s$  just above flatband [5][6].

The above expression (1) is for the total silicon charge. To analyze the dc current conduction through the inversion layer in the channel, and to analyze the terminal charges needed for large-signal and small-signal modeling, we need individual expressions for the bulk charge  $Q_B'$  and the inversion charge  $Q_I'$ . In the book  $Q_B' = -\gamma C_{ox} \sqrt{\psi_s}$  (4.3.12) is assumed, but this is a reasonable approximation only for  $\psi_s$  somewhat greater than zero. Noting that the inversion charge in (1) stems from the  $e^{\psi_s/\phi_t}$  term, and that the other components of the last term in parentheses under the second square root in (1) are negligible in all regions of operation, then the bulk charge can be identified as [7]

$$(2) \quad Q_B' = -\text{sgn}(\psi_s) \sqrt{2q\epsilon_s N_A} \sqrt{\phi_t e^{-\psi_s/\phi_t} + \psi_s - \phi_t}$$

whence from (2.6.2)

$$(3) \quad Q_I' = Q_C' - Q_B'.$$

We note that the semiconductor charge is also given by (see (2.4.23))

$$(4) \quad Q_C' = -C_{ox}' (V_{GB} - V_{FB} - \psi_s).$$

The approximation (2) is reasonable in all regions of operation. In particular, unlike (4.3.12) it is valid through flatband and into accumulation. In accumulation  $\psi_s < 0$ , the  $e^{-\psi_s/\phi_t}$  term in (2) dominates, and  $Q_B'$  is positive, comprising mobile accumulation holes.  $Q_B'$  is therefore no longer only a depletion charge, as it is for operation from somewhat above flatband, in depletion, through strong inversion. Hence our calling it ‘‘bulk charge’’ here rather than just depletion charge, as is done in the book.

The practical difficulty in applying the above expressions directly is that evaluation both of  $I_{DS}$  and of charges requires integration of (2) along the channel, but there is no closed form solution for this integral. Adoption of the approximation (4.3.12) enables a closed form solution for  $I_{DS}$  in inversion, but this approximation is not valid for charges in accumulation and the bottom portion of depletion. As discussed in Sec. 4.4, it is reasonable to use the first two terms of a Taylor expansion of  $Q_B'(\psi_s)$  as a basis for developing a model. It turns out that this is practical, accurate, and applies for charge computation as well as for modeling of  $I_{DS}$ . As discussed in Sec. 4.4 several choices of  $\psi_s$  can, and have been, selected as the point about which to apply the Taylor expansion, and for preservation of symmetry, simplicity of final equations, accuracy, and applicability to all regions of operation, the midpoint potential

$$(5) \quad \psi_{sm} = \frac{\psi_{s0} + \psi_{sL}}{2},$$

as first proposed in [1], turns out to be a good, indeed the best known, choice.

Here we will apply such an expansion, called *symmetric linearization* [1], to the bulk charge model (2) and derive expressions to model  $I_{DS}$  and all terminal charges in all regions of operation.

## Symmetric Linearization of Bulk Charge

To simplify notation, we will denote the surface potential relative to  $\psi_{sm}$  as

$$(6) \quad \psi_{sr} = \psi_s - \psi_{sm}$$

and introducing

$$(7) \quad \Delta\psi_s = \psi_{sL} - \psi_{s0}$$

then  $\psi_{sr}$  varies from  $-\Delta\psi_s/2$  at the source end of the channel to  $\Delta\psi_s/2$  at the drain end. Linearization of (2) about  $\psi_{sm}$  then gives

$$(8) \quad Q'_B(\psi_{sr}) = Q'_B(\psi_{sm}) + \left( \frac{\partial Q'_B}{\partial \psi_s} \right)_{\psi_s = \psi_{sm}} \psi_{sr}$$

and introducing (2.4.26a), the body effect coefficient  $\gamma = \sqrt{2q\epsilon_s N_A} / C'_{ox}$ , and the bulk charge linearization factor

$$(9) \quad \alpha_m = 1 + \frac{\gamma(1 - e^{-\psi_{sm}/\phi_t})}{2\sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t}}$$

(note that L'Hôpital's rule is needed to evaluate this at  $\psi_{sm} = 0$ , although for operation from accumulation through depletion the linearization is not really needed as  $\psi_s$  is uniform along the channel, depending only on  $V_{GB}$  and not on  $V_{CB}$ ) gives

$$(10) \quad Q'_B(\psi_{sr}) = -\text{sgn}(\psi_{sm})C'_{ox} \left( \gamma\sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + (\alpha_m - 1)\psi_{sr} \right).$$

Introducing this and (4) into (3) gives the linearized inversion charge

$$(11) \quad Q'_I = -C'_{ox} \left( V_{GB} - V_{FB} - \psi_{sm} - \gamma\sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} - \alpha_m \psi_{sr} \right)$$

where we have dropped the  $\text{sgn}(\psi_{sm})$  term as this expression is only applicable in inversion when  $\psi_{sm} > \phi_F$ . Note that the first four terms in parentheses are constant, independent of position along the channel, and only the final term  $\psi_{sr}$  varies along the channel. These first four terms represent the midpoint inversion charge in the channel, scaled by  $-C'_{ox}$  (it is not precisely an "average" normalized inversion charge, but as we will see represents the effect inversion charge that, when multiplied by the difference in surface potential across the channel, supports the drift component of drain-source current).

## Drain Current Model

Using (11) in (4.3.6) gives, with a variable substitution from  $\psi_s$  to  $\psi_{sr}$ ,

$$(12) \quad I_{DS} = \frac{W}{L} \mu C'_{ox} \int_{-\Delta\psi_s/2}^{\Delta\psi_s/2} \left( V_{GB} - V_{FB} - \psi_{sm} - \gamma\sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} - \alpha_m \psi_{sr} \right) d\psi_{sr} + \frac{W}{L} \mu \phi_t \left( Q'_I(\psi_{sL}) - Q'_I(\psi_{s0}) \right)$$

and from odd symmetry the integral of the term in  $\psi_{sr}$  goes to zero, and because the first four terms in (11) are independent of position along the channel they do not contribute to the diffusion current component, which is driven by the difference in inversion charge densities between drain and source ends of the channel and so is only affected by the  $\psi_{sr}$  term in (11), therefore

$$(13) \quad I_{DS} = \frac{W}{L} \mu C_{ox}' \left( V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + \alpha_m \phi_t \right) \Delta \psi_s$$

which is (4.4.10) with the more accurate all-region bulk charge model (2) in place of the model (4.3.12) (which, we stress, is only applicable for  $V_{GS}$  more than several  $\phi_t$  above flatband). The first 4 terms in parentheses in (13) are associated with the effective inversion charge level in the channel, and hence with the drift component  $I_{DS1}$  of  $I_{DS}$ , and the 5<sup>th</sup> term represents the diffusion component  $I_{DS2}$ . Note the inherent symmetry in (13); the term in parentheses depends only on  $\psi_{sm} = 0.5(\psi_{s0} + \psi_{sL})$  and flipping source and drain changes only the sign of the  $\Delta \psi_s$  term.

### Surface Potential as a Function of Position

To evaluate the charges, both for the change of variable needed for the integrals and also for the so-called Ward-Dutton partitioning of the channel charge into separate source and drain components [8], we need to know  $\psi_s(x)$ . Conveniently, the symmetric linearization procedure leads to an explicit and analytically tractable solution for surface potential as a function of position. The midpoint potential  $\psi_{sm}$  does not occur at the physical midpoint of the channel (except when  $V_{DS} = 0$ ). Assume it occurs at a position  $x_m$  along the channel, then integrating (4.3.4), with  $Q_I'$  from (11), from  $x_m$  (where  $\psi_{sr} = 0$ ) to  $x$  gives

$$(14) \quad I_{DS}(x - x_m) = W \mu C_{ox}' \left[ \left( V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + \alpha_m \phi_t \right) \psi_{sr} - 0.5 \alpha_m \psi_{sr}^2 \right]$$

Because the integration leading to (14) is no longer odd symmetric the term in the integrand of the drift current in  $\psi_{sr}$  remains, and does not vanish as it does when evaluating (13). Introducing  $I_{DS}$  from (13) gives, after some rearrangement,

$$(15) \quad x = x_m + \frac{L}{\Delta \psi_s} \left( \psi_{sr} - \frac{\psi_{sr}^2}{2H} \right)$$

where

$$(16) \quad H = \phi_t + \frac{V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t}}{\alpha_m}.$$

Eq. (15) is a quadratic equation can be explicitly solved for  $\psi_{sr}(x)$ , and using (6) gives

$$(17) \quad \psi_s(x) = \psi_{sm} + H \left( 1 - \sqrt{1 - \frac{2\Delta \psi_s(x - x_m)}{HL}} \right),$$

but more important, directly (15) gives

$$(18) \quad \frac{dx}{d\psi_s} = \frac{dx}{d\psi_{sr}} = \frac{L}{\Delta \psi_s} \left( 1 - \frac{\psi_{sr}}{H} \right)$$

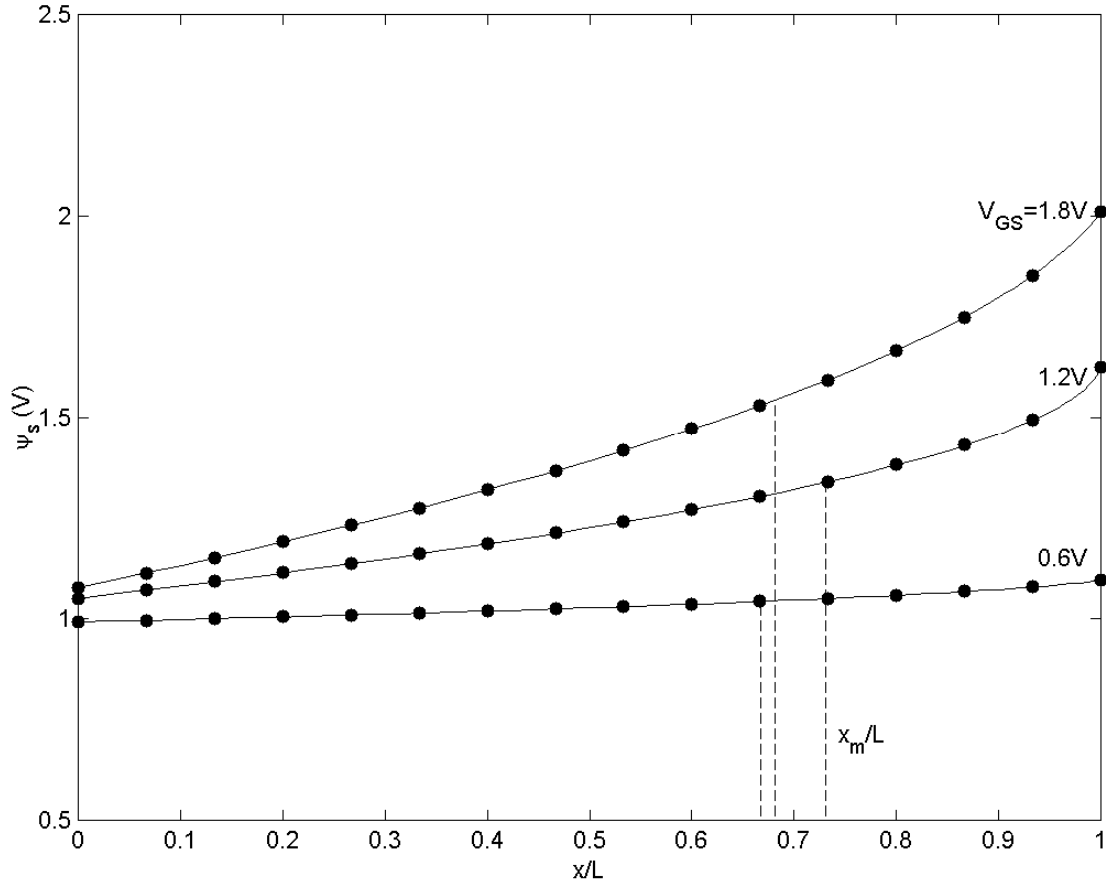
which will be required for changing the variable of integration when evaluating charges.

Evaluating (15) at the source end of the channel, where  $x = 0$  and  $\psi_{sr} = -\Delta \psi_s/2$ , and rearranging gives

$$(19) \quad x_m = \frac{L}{2} \left( 1 + \frac{\Delta \psi_s}{4H} \right).$$

The potential midpoint is therefore at the physical midpoint of the channel when  $\psi_{sL} = \psi_{s0}$ , which only holds when  $V_{DS} = 0$ , and moves toward the drain end of the channel for  $V_{DS} > 0$ , when  $\psi_{sL} > \psi_{s0}$ .

The surface potential vs. position given by (17) is valid for all bias regimes; however, from accumulation through the bottom range of moderate inversion  $\psi_s$  is essentially independent of  $V_{CB}$ , it depends only on  $V_{GB}$ , and so does not vary significantly along the channel. Fig. 1 compares surface potential vs. position, in the inversion region of operation, from the complete all-region model of the book and from the symmetric linearization model. There is no discernable difference.



**Fig. 1** Comparison of surface potential vs. distance from the source for the complete all-region model of the book (lines) and the symmetric linearization model (symbols). The vertical dashed lines show the position of the midpoint potential  $\psi_{sm}$ .  $W/L = 10\mu\text{m}/10\mu\text{m}$ ,  $V_{FB} = -0.8\text{V}$ ,  $N_A = 5 \times 10^{17}\text{cm}^{-3}$ ,  $t_{ox} = 2.5\text{nm}$ ,  $V_{SB} = 0$ , and  $V_{DS} = 1\text{V}$ .

### Terminal Charge Models

From (6.2.3a), using (11) and changing the variable of integration from  $x$  to  $\psi_{sr}$  using (6) and (18),

$$(20) \quad Q_I = W \int_0^L Q'_I dx = -\frac{LWC'_{ox}}{\Delta\psi_s} \int_{-\Delta\psi_s/2}^{\Delta\psi_s/2} \left( V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} - \alpha_m \psi_{sr} \right) \left( 1 - \frac{\psi_{sr}}{H} \right) d\psi_{sr}$$

which, noting that because of odd symmetry the integrals of terms that are odd powers of  $\psi_{sr}$  vanish, gives

$$(21) \quad Q_I = -LWC'_{ox} \left( V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + \frac{\alpha_m \Delta\psi_s^2}{12H} \right)$$

(again recall that the inversion charge is zero for  $\psi_{sm} < \phi_F$  so we do not include the  $\text{sgn}(\psi_{sm})$  term). From (6.2.3b), using (2.3.4), (4), and the change of the variable of integration from  $x$  to  $\psi_{sr}$ ,

$$\begin{aligned}
(22) \quad Q_G &= W \int_0^L Q'_G dx = \frac{LWC'_{ox}}{\Delta\psi_s} \int_{-\Delta\psi_s/2}^{\Delta\psi_s/2} (V_{GB} - V_{FB} - \psi_{sm} - \psi_{sr}) \left(1 - \frac{\psi_{sr}}{H}\right) d\psi_{sr} - Q_o \\
&= LWC'_{ox} \left( V_{GB} - V_{FB} - \psi_{sm} + \frac{\Delta\psi_s^2}{12H} \right) - Q_o
\end{aligned}$$

where again terms in odd powers of  $\psi_{sr}$  in the integral vanish. Similarly, From (6.2.3c), using (10) and changing the variable of integration from  $x$  to  $\psi_{sr}$  using (6) and (18),

$$(23) \quad Q_B = W \int_0^L Q'_B dx = -\text{sgn}(\psi_{sm}) \frac{LWC'_{ox}}{\Delta\psi_s} \int_{-\Delta\psi_s/2}^{\Delta\psi_s/2} \left( \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + (\alpha_m - 1)\psi_{sr} \right) \left(1 - \frac{\psi_{sr}}{H}\right) d\psi_{sr}$$

giving, dropping odd powers of  $\psi_{sr}$  in the integral,

$$(24) \quad Q_B = -\text{sgn}(\psi_{sm}) LWC'_{ox} \left( \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} - \frac{(\alpha_m - 1)\Delta\psi_s^2}{12H} \right).$$

Note that this expression includes the accumulation charge (represented by the exponential term) and so is valid in all regions of operation, not just inversion and the upper part of depletion.  $Q_B$  could also have been calculated, from charge balance, as  $Q_B = -Q_I - Q_G - Q_o$ .

To compute the drain charge we need to use the partitioning scheme of (6.3.9a),

$$(25) \quad Q_D = W \int_0^L \frac{x}{L} Q'_I dx$$

and from (15) and (19)

$$(26) \quad \frac{x}{L} = \frac{1}{2} \left( 1 + \frac{\Delta\psi_s}{4H} \right) + \frac{1}{\Delta\psi_s} \left( \psi_{sr} - \frac{\psi_{sr}^2}{2H} \right)$$

so using this with (11) and proceeding as before

$$\begin{aligned}
(27) \quad Q_D &= -\frac{LWC'_{ox}}{2\Delta\psi_s} \int_{-\Delta\psi_s/2}^{\Delta\psi_s/2} \left\{ \left( V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} - \alpha_m \psi_{sr} \right) \right. \\
&\quad \times \left[ 1 + \frac{\Delta\psi_s}{4H} + \frac{2}{\Delta\psi_s} \left( \psi_{sr} - \frac{\psi_{sr}^2}{2H} \right) \right] \left( 1 - \frac{\psi_{sr}}{H} \right) \left. \right\} d\psi_{sr} \\
&= -\frac{LWC'_{ox}}{2} \left[ V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} - \frac{\alpha_m \Delta\psi_s}{6} \left( 1 - \frac{\Delta\psi_s}{2H} - \frac{\Delta\psi_s^2}{20H^2} \right) \right]
\end{aligned}$$

where, serendipitously, quite a few terms cancel out, and subtraction gives  $Q_S = Q_I - Q_D$  as

$$(28) \quad Q_S = -\frac{LWC'_{ox}}{2} \left[ V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + \frac{\alpha_m \Delta\psi_s}{6} \left( 1 + \frac{\Delta\psi_s}{2H} - \frac{\Delta\psi_s^2}{20H^2} \right) \right].$$

The above expressions for the charges are substantially simpler than those based on analysis without the symmetric linearization procedure (8) (see Prob. 6.8) but give nearly identical results. They are also more numerically stable to evaluate.

## Robust Numerical Evaluation

The term  $V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t}$  appears in the above expressions for the inversion charges (drain, source, and total). Outside of strong inversion this term involves the difference of two nearly equal numbers, so is not numerically robust to evaluate. We now present a numerically robust procedure to evaluate this quantity [1][9]. As the inversion charge is relevant only for operation above depletion we take  $\psi_{sm}$  to be positive.

From (3.2.11), at the midpoint potential we have

$$(29) \quad V_{GB} - V_{FB} - \psi_{sm} = \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right)$$

where  $V_{mB}$  is the channel-to-bulk potential (technically the splitting of the electron and hole quasi-Fermi levels between the surface and deep in the bulk, see Fig. 3.4c), which at present we do not know but will shortly show how to compute. Using this relation gives

$$(30) \quad \begin{aligned} & V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} \\ &= \gamma \left[ \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right) - \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} \right] \end{aligned}$$

The right hand side of this expression is of the form  $\sqrt{a + \xi} - \sqrt{a}$  where, outside of strong inversion,  $\xi$  is small compared to  $a$ . As noted in the footnote on p. 90, this can be rewritten as the equivalent form  $\xi / (\sqrt{a + \xi} + \sqrt{a})$  which does not rely on the difference between two quantities of similar magnitude. Applying this reformulation to (30) gives

$$(31) \quad \begin{aligned} & V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} \\ &= \frac{\gamma e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right)}{\sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t} + e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right) + \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t}} \end{aligned}$$

which allows numerically stable computation of  $V_{GB} - V_{FB} - \psi_{sm} - \gamma \sqrt{\phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t}$ .

We now need to compute  $V_{mB}$ , or more practically  $e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right)$ . We can rewrite (29) as

$$(32) \quad \phi_t e^{-\psi_{sm}/\phi_t} + \psi_{sm} - \phi_t + e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right) = \frac{(V_{GB} - V_{FB} - \psi_{sm})^2}{\gamma^2}$$

and similar forms hold for both the source end of the channel, with  $\psi_{sm}$  and  $V_{mB}$  replaced by  $\psi_{s0}$  and  $V_{SB}$ , respectively, and at the drain end of the channel, with  $\psi_{sm}$  and  $V_{mB}$  replaced by  $\psi_{sL}$  and  $V_{DB}$ , respectively. Forming the difference between (32) and its equivalent form at the source end of the channel gives, after some manipulation,

$$(33) \quad \begin{aligned} & e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right) \\ &= e^{-(2\phi_F + V_{SB})/\phi_t} \left( \phi_t e^{\psi_{s0}/\phi_t} - \psi_{s0} - \phi_t \right) + \phi_t \left( e^{-\psi_{s0}/\phi_t} - e^{-\psi_{sm}/\phi_t} \right) - \frac{\Delta\psi_s}{2} - \frac{\Delta\psi_s (V_{GB} - V_{FB} + \Delta\psi_s/4)}{\gamma^2} \end{aligned}$$

and forming the difference between (32) and its equivalent form at the drain end of the channel gives

$$\begin{aligned}
& e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right) \\
(34) \quad & = e^{-(2\phi_F + V_{DB})/\phi_t} \left( \phi_t e^{\psi_{sL}/\phi_t} - \psi_{sL} - \phi_t \right) + \phi_t \left( e^{-\psi_{sL}/\phi_t} - e^{-\psi_{sm}/\phi_t} \right) + \frac{\Delta\psi_s}{2} + \frac{\Delta\psi_s (V_{GB} - V_{FB} - \Delta\psi_s/4)}{\gamma^2}
\end{aligned}$$

and averaging (33) and (34), expanding the exponential terms involving the negatives of the surface potentials to second order (recall we are only concerned about modeling the inversion charge, where  $\psi_s > \phi_F$  and this expansion is accurate) gives

$$\begin{aligned}
& e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right) \\
(35) \quad & = \frac{e^{-(2\phi_F + V_{SB})/\phi_t} \left( \phi_t e^{\psi_0/\phi_t} - \psi_{s0} - \phi_t \right) + e^{-(2\phi_F + V_{DB})/\phi_t} \left( \phi_t e^{\psi_{sL}/\phi_t} - \psi_{sL} - \phi_t \right)}{2} - \Delta\psi_s^2 \left( \frac{1}{4\gamma^2} - \frac{e^{-\psi_{sm}/\phi_t}}{8\phi_t} \right)
\end{aligned}$$

which provides a numerically stable method to evaluate  $e^{-(2\phi_F + V_{mB})/\phi_t} \left( \phi_t e^{\psi_{sm}/\phi_t} - \psi_{sm} - \phi_t \right)$  in (31).

## References

- [1] T.-L. Chen and G. Gildenblat, "Symmetric bulk charge linearisation in charge-sheet MOSFET model," *Electronics Letters*, vol. 37, no. 12, pp. 791-793, Jun. 2001.
- [2] G. Gildenblat, W. Wu, X. Li, R. van Langevelde, A. J. Scholten, G. D. J. Smit, and D. B. M. Klaassen, "Surface-potential-based compact model of bulk MOSFET," in *Compact Modeling: Principles, Techniques and Applications*, G. Gildenblat (Ed), Springer, pp. 3-40, 2010.
- [3] H. C. Pao and C. T. Sah, "Effects of diffusion current on characteristics of metal-oxide (insulator)-semiconductor transistors," *Solid State Electron.*, vol. 9, no. 10, pp. 927-937, Oct. 1966.
- [4] Y. Tsividis, *Operation and Modeling of the MOS Transistor*, 2<sup>nd</sup> ed., McGraw-Hill, 1999.
- [5] C. C. McAndrew and J. Victory, "Accuracy of Approximations in MOSFET Charge Models," *IEEE Trans. Electron Devices*, vol. 49, no. 1, pp. 72-81, Jan. 2002.
- [6] W. Wu, T.-L. Chen, G. Gildenblat, and C. C. McAndrew, "Physics-based mathematical conditioning of the MOSFET surface potential equation," *IEEE Trans. Electron Dev.*, vol. 51, no. 7, pp. 1196-1200, Jul. 2004.
- [7] J. Victory, C. C. McAndrew, and K. Gullapalli, "A time-dependent, surface potential based compact model for MOS capacitors," *IEEE Electron Device Letters*, vol. 22, no. 5, pp. 245-247, May 2001.
- [8] D. E. Ward, *Charge-based modeling of capacitance in MOS transistors*. Technical Report G201-11, Integrated Circuits Laboratory, Stanford University, June 1981.
- [9] H. Wang, T.-L. Chen, and G. Gildenblat, "Quasi-static and nonquasi-static compact MOSFET models based on symmetric linearization of the bulk and inversion charges," *IEEE Transactions on Electron Devices*, vol. 50, no. 11, pp. 2262-2272, November 2003.